

# Exponential Tethers for Accelerated Space Elevator Deployment\*

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## Abstract

An exponential space elevator is a space elevator with a tether cross-section that varies exponentially with altitude. With such an elevator it is possible to reel in tether material at one end of the elevator while reeling out at the other end, without changing the overall taper profile. I show how to use this property to build up or clone a space elevator much more efficiently than with standard climber-based methods.

## Introduction

Space elevators are a promising candidate for replacing rockets as the principal means of transportation into space. With them, space can be reached by climbing a giant tether, attached to the Earth at one end, and held in place by centrifugal force due to the Earth's rotation. The concept was first proposed in Russian by Artsutanov [1, 2], and later introduced in English by Pearson [3]. The idea was then mainly developed by science-fiction authors, including Arthur C. Clarke [4], who said "The space elevator will be built about 50 years after everyone stops laughing". The laughing has largely stopped since a proposed light weight elevator concept by Edwards [5].

In his design, Edwards proposes to use carbon nanotube composites as the building material for the elevator, the availability of sufficiently strong composites currently being the main impediment to building the elevator. In his proposal, the elevator construction begins with the launch to geosynchronous orbit (GEO) of a very light initial tether; light enough for existing launch technology. Once at GEO, this initial tether is deployed down to an *anchor* station on the surface of the Earth. At the other end a *counterweight* located beyond GEO pulls on the tether to keep it upright and in tension. This initial tether is

light and therefore has a small payload. It is nevertheless strong enough for a light *climber* vehicle to pull itself up the tether. The tether is ribbon shaped to allow the climber to climb up it, and to protect it from space debris. As it climbs, the climber uses a spool of tether material it is carrying to slightly widen the existing cable. This climber is followed by many others, each one helping to build up the cable a little more. After a couple of hundred climbers, the space elevator is complete, ready to lift much larger payloads.

Edwards, like his predecessors, considers that the tether has a cross-section that depends on altitude. Indeed, to maximize the payload, the cross-section of the tether should be chosen so that it is fully loaded along its full length. Indeed, if some parts of the tether were not fully loaded, material could be removed from those parts, resulting in a lighter elevator with a greater lift capacity. We shall refer to these tethers as uniform-stress tethers. They are skinny at the surface of the Earth, increase in cross-section with altitude up to GEO, after which they decrease in cross-section. They are the uncontested choice for making a space elevator that lifts large payloads, and they tend to be regarded as the only interesting tethers for space elevators.

My goal in this paper is to show that this idea is wrong, and that other tether profiles are also useful for space elevators. Indeed, while uniform stress-tethers are ideal for lifting payloads, they are not necessarily the best tether profile to use while building up the elevator. Indeed, the build up of a space elevator is limited by how fast one can get mass off the ground onto the elevator. During build-up, the mass that needs to be lifted is essentially made up of ultra-strong tether material. This is very different from the type of mass that is to be lifted once the elevator is in general use (satellites, probes, ...). In Edwards' proposal, climbers are used to lift both types of mass. But while the tether is on the climber, it is just dead weight; its strength is wasted.

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In this paper, instead of lifting ribbon materials using climbers, we shall lift ribbon material by adding material to the bottom of the elevator. Think of a kite. If you want to lift string off the ground, you can either have a roll of string lifted along the kite’s string with a little climber device, or you can simply let out more string; the kite will rise to maintain tension in the string. In the latter case, as material is lifted, it is also providing strength to help lift the material below it. When this principle is applied to space elevators, we shall see that mass can be lifted much faster than is possible using climbers.

Unfortunately, if the space elevator has a uniform-stress profile, then this method will not work. Indeed, the thick part of the elevator that is initially at GEO ends up past GEO, and the elevator no longer has uniform stress. Moreover, it is not clear what profile should be given to the new material being added at the base of the elevator. Consequently, we shall use exponential tethers instead of uniform stress tethers. With these tethers, the cross-section depends exponentially on altitude. When the tether is translated up or down, the cross-section of the elevator is simply multiplied by a constant factor; the overall profile remains unchanged. Exponential taper is the natural taper for tethers that get translated.

The idea of using an exponential tether that gets translated upwards during buildup is not entirely new. To my knowledge it was first proposed by Cline [6]. The *reel-to-reel* buildup method I describe is essentially the method proposed by Cline. The *breeder elevator* method is also hinted at in his work. In writing this paper I hope to expose a wider audience to these excellent ideas. I have also tried to go beyond Cline’s work. In particular, I feel that I have provided quantitative arguments in favor of these buildup methods, I have studied them over a wide range of elevator parameters, I have recognized that exponential taper is the canonical taper profile to use, and I have realized that the breeder elevator can be used even when the tether material is too weak to permit tethers with inverse taper. The *pull-down* and *redeploy and splice* methods are also, new to my knowledge.

In the remainder of this paper, I shall first consider the basic equations that apply to exponential tethers, and review some of their properties in Section 1. Then I shall present different ways in which exponential tethers can be used in space elevator con-

struction in Section 2. Finally, I will compare the proposed schemes with Edwards’ climber-based scheme in Section 3.

## 1 Properties

Before looking at how exponential tethers can aid in space elevator deployment, we look at some of the properties of exponential tethers. For simplicity, in this paper we will consider that the tether has no elasticity and that it is located in the equatorial plane. All calculations will be done assuming an Earth elevator (radius  $r_e = 6.38 \cdot 10^6$  m, mass  $M_e = 5.98 \cdot 10^{24}$  kg). We will explore a range of tether materials. Most generally, tether material is characterized by its strength to weight ratio. However, for ease of understanding, I will characterize a tether material by its maximum tensile strength (after safety factor), and keep the density fixed at  $1300$  kg/m<sup>3</sup>.

### 1.1 Taper Profile

An exponential tether has a cross-section with an exponential dependence on position along the tether:

$$A(r) = A_0 e^{\gamma r} \tag{1}$$

In this expression  $r$  is the distance to the center of the planet,  $\gamma$  is the exponential growth parameter of the tether, and  $A_0$  is the cross-section that the tether would have if it was extended to the center of the Earth.

The key property of exponential tethers is that their cross-section increases uniformly by a factor  $e^{\gamma d}$  when they are translated a distance  $d$ . This makes them the tether of choice for buildup methods like the ones we will introduce in Section 2.

The value of  $\gamma$  determines how much the elevator tapers. When it is positive, the elevator broadens with altitude, this is *normal taper*. When it is negative, the elevator gets narrower with altitude, this is *inverse taper*. In the space elevator literature, people usually consider the taper ratio, which is the ratio of the elevator cross-section at GEO to its cross-section

at the anchor.<sup>1</sup> The taper ratio is given by

$$\beta = e^{\gamma(r_g - r_e)} \quad (2)$$

where  $r_g$  is the distance from the center of the Earth to the GEO. I shall present data in terms of taper ratio, as it is easier to grasp than the growth parameter.

## 1.2 Tension and Stress in the Tether

For the tether to be in equilibrium it must satisfy Newton's second law all along its length. This gives an equation for the tension in the tether:

$$\frac{dT}{dr}(r) = -\rho A(r)g(r) \quad (3)$$

In this equation  $T$  is the tension in the tether, and  $g$  is the gravitational field (negative when it is pointing towards the Earth). The gravitational field incorporates gravity and a centrifugal term due to the Earth's rotation:

$$g(r) = -\frac{GM_e}{r^2} + r\Omega^2 \quad (4)$$

Here  $\Omega$  is the angular velocity of the Earth's rotation and  $G$  is the gravitational constant. The gravitational field changes sign at GEO, at  $r_g = (GM_p/\Omega^2)^{1/3}$ . Therefore, the tension is maximum at that altitude. Equation (3) can now be integrated. However, the solution includes the integral exponential function and is not very insightful.

To completely know the tension in the tether, we need to specify some boundary conditions. At the anchor point, we simply need  $T(r_e) \geq 0$  so that the tether is in tension. At the counterweight, the tension in the tether must be exactly the tension needed to counteract the gravitational force (i.e., here mainly the centrifugal force) on the counterweight  $T(r_c) = M_c g(r_c)$ . Because of this condition, the counterweight must be beyond GEO. This makes sense, the elevator has to reach past GEO for the centrifugal force on the elevator to counterbalance the Earth's gravitational field. These boundary conditions will be incompatible if the counterweight's mass is insufficient.

<sup>1</sup>For uniform-stress elevators, the cross-section at GEO is also the maximum cross-section of the elevator. For exponential elevators this is no longer the case, so the taper ratio is not the ratio of maximum cross-section to cross-section at the base of the elevator.

The stress  $\sigma$  in the tether can be expressed in terms of the tension by  $\sigma = T/A$ . We get an equation for  $\sigma$  from (1) and (3):

$$\sigma(r)\gamma + \frac{d\sigma}{dr}(r) = -\rho g(r) \quad (5)$$

This equation shows that exponential tethers do not usually have their maximum stress at the synchronous altitude. The maximum stress is reached at the anchor, at the counterweight, or at a point where  $\frac{d\sigma}{dr}(r) = 0$ . In the latter case, the maximum stress  $\sigma_m$  and the altitude  $r_m$  at which it is reached are related by

$$\sigma_m \gamma = -\rho g(r_m) \quad (6)$$

Taking the derivative of (5) at a point where (6) holds, we find that there are no local minima of the stress. Therefore, the maximum stress is reached at a single point along the tether. Moreover, we note that as  $\sigma_m$  increases,  $r_m$  gets further from GEO. We are now ready for a theorem that greatly aids in understanding the range of altitudes that can be reached by an exponential tether. The proof of this theorem is left up to the reader.

**Theorem 1.** *If an exponential tether, with one extremity on each side of GEO, exceeds a stress  $\sigma_0$ , then it exceeds it at one of its extremities or at  $r_0 = g^{-1}(\sigma\gamma/\rho)$  (the last case only applies if  $r_0$  is within the length of the tether).*

## 1.3 Critical Strength

In Section 2, a number of the applications of exponential tethers need the tether to have inverse taper to work. Inverse taper is only possible for sufficiently strong tethers. To find the critical strength  $\sigma_c$  beyond which inverse taper is possible, we consider an untapered tether with  $T = 0$  at the anchor. In that case the stress is maximum at GEO, from (6). We calculate the tension at GEO by integrating (3) with  $\gamma = 0$ . The stress at GEO is equal to the critical stress:

$$\sigma_c = \rho GM_e \left( \frac{1}{r_e} - \frac{1}{r_g} \right) + \frac{1}{2} \rho \Omega^2 (r_e^2 - r_g^2) \quad (7)$$

For the Earth, the critical strength is  $\sigma_c = 63.0$  GPa. This is less than the strength that is assumed by Edwards [5] for his designs.

## 1.4 Allowable Counterweight Altitudes, Taper Ratios and Tether Strengths

With uniform stress tethers, for a given strength tether, it is possible to reach any altitude, and there is a unique way of doing so. Reaching altitudes far above or below GEO will just have a high cost, measured by a large taper ratio. With exponential tethers, the situation is much more complex, which comes from the three places where the stress  $\sigma_0$  can be exceeded in Theorem 1.

Figure 1 shows the tether lengths that can be achieved for an Earth space elevator with various strengths and taper ratios.<sup>2</sup> This plot was generated by considering, for a given taper ratio and elevator length, the maximum stress that is achieved by integrating (3) from each end of the elevator, starting with a tension of zero. The bottom branch of each plot corresponds to a limit where the tension is zero at the anchor. The top branches correspond to limits where the tension is zero at the counterweight. For a given tether length, there is a minimum necessary tether strength. That minimum strength is at the intersection of the two branches.

## 2 Deployment Scenarios

We shall now look at ways in which exponential tethers can be used to build up a space elevator. First we shall look at *reel-to-reel* buildup in which a tether can be deployed by reeling in material at the counterweight and letting material out at the anchor. We will then discuss an alternative method, *the pull-down* build up, where both reels are on the ground and the counterweight is a simple pulley. Next we will see how the material that is accumulated at the counterweight could be used to build a second elevator. This will lead to the idea of a *breeder* which can be used to rapidly produce two similarly sized elevators from one. Finally, by merging the two resulting elevators one elevator can be caused to nearly double in cross-section in a single step, leading to vastly improved tether growth rates; this is the *redeploy and splice*

<sup>2</sup>If elasticity were taken into account then the elevator would be unstable when we are too near the equator or the counterweight is too light. Taking this instability into account would force us to slightly increase the necessary strengths we find in this analysis.

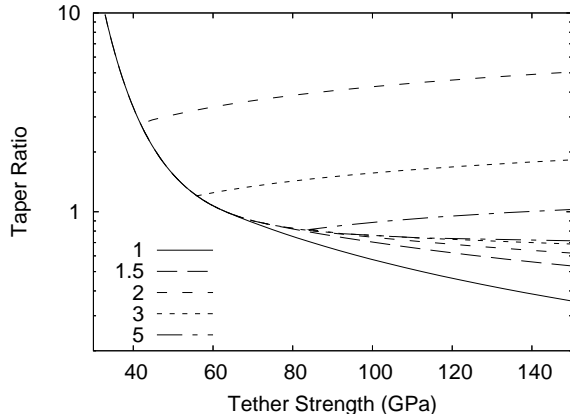


Figure 1: Acceptable combinations of counterweight position, tether strength and taper ratio. Each line corresponds to a counterweight position (specified by  $r_c/r_g$ ), which can be reached in the region on the right of the line.

buildup method.

### 2.1 Reel-to-Reel Buildup

Reel-to-reel buildup is the most natural way to use an exponential tether. It requires a tether with inverse taper, and thus the tether strength must exceed the critical strength (see Section 1.3). The buildup method is as follows (see also Figure 2):

1. An initial exponential tether is deployed.
2. The initial tether is built up by reeling in material at the counterweight while paying out material at the anchor. Because of the inverted taper, the tether cross-section slowly increases until it reaches the desired cross-section for the final elevator.
3. A uniform-stress tether is reeled up, and the elevator is ready to lift payloads.

#### 2.1.1 Advantages

**Speed** For sufficiently strong tethers, (i.e., when the inverted taper becomes significant) this method is much faster than climber based buildup (see Section 3).

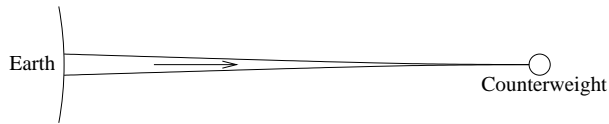


Figure 2: A reel-to-reel elevator being built up.

**Simplicity** With this method, the only moving part is the spool at the counterweight. This is a great simplification over having to send hundreds of climbers up the tether.

**Quality of ribbon** There is no high altitude ribbon splicing, unlike climber based buildup. This may allow a lower safety factor for reel-to-reel buildup.

**Ease of repair** Ribbon repairs can be performed on the ground by reeling the uniform-stress section of the ribbon down, and then reeling it back up repaired. Such repairs could be done at regular intervals and would require a few weeks of reeling time. The repairs could be done as the tether is reeled down, or the lowered section of the tether could be replaced by a new tether section and the old one decommissioned or repaired at leisure.

### 2.1.2 Engineering Issues

**No Counterweight Growth** This method makes the tether grow in a uniform manner. However, the counterweight does not change during buildup, except for the addition of extra tether material. Thus the initial counterweight has to be sized for the final size of the elevator. In particular, the reel of the initial counterweight must be sized to accommodate all the ribbon that is to be deployed, and the structure of the initial counterweight must withstand the full counterweight tension of the final elevator. This means extra mass for the initial counterweight. This limitation can be mitigated by building up the elevator in stages: first build up the elevator to the limit of the initial counterweight; then pull up a uniform-stress tether; use it to lift a new larger counterweight; pull the uniform-stress tether section back down; finally, restart the reel-to-reel buildup; repeat as necessary.

**Location of Power Consumption** With climber-based buildup, most of the power is spent near the surface of the Earth, closest to the power beaming stations. With reel-to-reel buildup the power is all consumed in the reeling motor at the counterweight. Power has to be beamed farther, but at a fixed target. Alternatively, solar panels could be used at the counterweight, but they have to be expanded as the elevator gets broader and the expended power increases. At the anchor point, power is being generated from the work of the tether tension.

**Waste of Ribbon Material** If the tether strength is not sufficiently large compared to the critical strength (i.e., the taper is too small) then the amount of tether material used to build the elevator gets multiplied by a large factor. Depending on the cost of the tether material this can greatly increase the total cost of the project. In many cases the extra tether material can be put to use. De facto, it is used as material for the counterweight (getting some other material to the counterweight would actually require extra effort and time). It can also be reused as elevator material in various ways, some of which shall be presented later in this section. Finally, it can be used by other space based construction projects that need high strength fibers [6].

**Excessive Inverse Taper of Uniform-Stress Tether** Uniform-stress tethers can be so narrow at the counterweight end that they cannot be pulled up without exceeding the tether strength. In practice this is not a significant issue; the bottom part of the final elevator can be uniform-stress to maximize the payload to GEO while the top part is exponential. This hybrid configuration would only make a difference for interplanetary payloads that benefit from riding the elevator as high as possible.

**Reliability of Counterweight** In reel-to-reel buildup, the reeling mechanism at the counterweight has to function flawlessly throughout the buildup process. This appears to be a weakness compared with hundreds of climbers that each need to work for only a couple of weeks, and for which recovery strategies have already been proposed [5]. Nevertheless, redundancy can be built into the counterweight, for example by having an independent reeling system that is initially detached from the ribbon, and that attaches

itself and takes over the reeling process if the primary system fails. Alternatively, we can try to move the reeling mechanism away from the counterweight as will be proposed in Section 2.2.

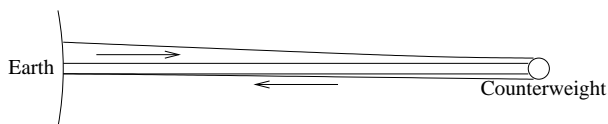


Figure 3: A pull-down elevator being built up.

**Wind, Atomic Oxygen and Space Debris** Because each section of cable travels from the anchor to the counterweight, each section encounters the whole range of perils that a space elevator must endure. During buildup, we can no longer use the strategy of having a thick but narrow tether within the atmosphere against wind loading, a tether designed to withstand chemical attack in the upper atmosphere, and a wider tether in LEO to resist damage from space debris [5]. The final uniform-stress tether can still have these properties, of course.

During buildup, the atomic oxygen problem is not an issue, as each length of tether only crosses the affected altitude range for a few hours. Wind and Space Debris, however, must be protected against at all times as they can cause immediate damage. Here the answer lies either in changing the tether geometry between the upper atmosphere and LEO (the tether could be initially rolled up laterally, and progressively unroll with altitude), or in designing the tether so that it has both a large total width and a small wind resistance (by having a ribbon made up of widely spaced relatively thick threads – this would not be as easy if the ribbon was built up from hundreds of smaller ribbons).

**Counterweight Altitude** As the tether is reeled in, the counterweight’s mass will change, as well as the tension applied to it, therefore the counterweight’s altitude may have to change during build up. This aspect deserves further consideration.

## 2.2 Pull-Down Buildup

A number of problems with reel-to-reel buildup can be solved by making a small modification to the scheme. Instead of spooling the tether up at the counterweight, it goes around a pulley, and returns to a second anchor point on Earth to be spooled up. Figure 3 illustrates the scheme.

### 2.2.1 Advantages

As before, the taper cross-section increases with time because of the inverse taper of the upwards strand of the elevator. A number of problems with the previous design are now solved, though:

**Location of Power Consumption** Now, the counterweight is a passive element. All the power for getting the tether up is provided directly by the ground station that is pulling on the down tether. In fact, if the counterweight is far enough beyond GEO, it is possible that the energy needed to pull the down strand is less than the work done by tension at the base of the up strand. In this case, build up would actually generate energy (taken from the Earth’s rotation). Whether this occurs for realistic elevator configurations requires further study.

**Reliability of Counterweight** Mechanically, the counterweight is greatly simplified as it is just a pulley. It can be designed with multiple bearings in series so that many bearings need to fail before the pulley ceases working.

**Waste of Ribbon Material** Since tether material is returned to Earth instead of being accumulated at the counterweight, it can be immediately and easily recycled. For example ribbon material arriving at the bottom of the down tether tether can be broadened and immediately sent back along the up tether.

**No Counterweight Growth** At first sight, this is more of a problem than before, as the counterweight is not building up mass in this scheme. In fact, the counterweight can be made to build up mass by cutting the ribbon in two at the counterweight. One part is reeled up by the counterweight, and contributes to growing the counterweight; the other goes into the down tether and allows the ground to provide the energy for the deployment. Moreover, be-

cause the down tether has the opposite taper from the up taper, it will be able to carry a significant payload (particularly for large cable strengths that have a lot of taper). Thus it will be possible to stop the elevator build up, and send a climber up the down tether to bring necessary structural elements to the counterweight. Unlike the simple reel-to-reel deployment, there would be no need to temporarily deploy a uniform-stress-tether, thus saving weeks of time.

### 2.2.2 Engineering Issues

**Getting Started** Initial deployment is simplest if the tether is deployed with two strands, ready to start reeling. Alternatively, a single strand can be deployed, and a climber can be used to pull the free end of the tether from the counterweight to the anchor, so that pull-down deployment can begin.

**Tangled Tethers** Because there are two tethers, there is a risk of tangling. Separating the two anchor locations should minimize this risk, but this strategy does not work when the tether is initially deployed. When the tethers are in motion, the Coriolis effect can also help.

## 2.3 Breeder Elevator

One common remark that is made about space elevators is that once the first one is built, the second one is easy. Indeed, the necessary parts for the second elevator can be cheaply lifted into GEO by the first elevator, without all the stringent mass limitations that apply for chemical rockets. In fact, building the second elevator should be a top priority to avoid having to start over from scratch if the first elevator was destroyed. Edwards makes this point [5] and suggests that producing up to ten cables in rapid succession may be wise.

Using climbers, a uniform-stress space elevator can clone itself in 210 days assuming the tether parameters from [5]. The simplest way of doing this is to build up the ribbon until it has doubled in width, and then split it down the middle (with some work needed to get half the climbers to go with each half of the ribbon). However, as we have seen, it is possible to get tether mass into orbit faster by reeling than by using climbers. Therefore, we can expect space elevator cloning to be faster by using exponential tether techniques.

Here is a proposed method for cloning an exponential elevator:

1. Reel up the tether material that is needed for the new elevator (enough to span the distance from Earth to counterweight and provide sufficient counterweight mass for the new elevator).
2. Cut the tether at counterweight, and attach it to a new spool.
3. Reel up a uniform-stress tether.
4. Send up the structure and machinery for the new elevator's counterweight.
5. Attach the free end of the tether on the old spool to the tether connected to Earth.
6. Reel down the uniform-stress tether.
7. Separate the two tethers at the anchor, move the new elevator to its assigned location.

At the end of this procedure there are two elevators of roughly the same size. For an elevator with inverse taper, the new elevator is larger than the initial one because in Step 1 the elevator is effectively being built up. It is not necessary, however, that the initial elevator have inverse taper. It is perfectly possible to clone an exponential taper that is narrower at its base by this method. In this case the new elevator would have a smaller cross-section than the old one, and could be built up with more reeling.

This method is good because it is fast. It takes into account the fact that tether material can be lifted faster than random hardware by using the reel-to-reel method. Random hardware would have to be lifted on climbers. To optimize the method, the mass of the structure and machinery for the new elevator has to be small. As much of the new elevator's mass as possible should be tether material. The total cloning time should be on the order of four to five times the time to reel a full anchor to counterweight length of ribbon, based on steps 1, 3, 4 and 5. This should take under 4 months with parameters similar to those in [5]. Most of this time is spent getting the new counterweight in place. If a nearby uniform-stress elevator is around it can be used to lift the new counterweight to GEO, after which the counterweight can climb the exponential tether to the old counterweight. Alternatively, multiple clonings can be done

at once, avoiding the need to deploy the uniform-stress tether more than once. Because of its ability to spawn new elevators at a high rate, I have called this elevator a *breeder elevator*.

One big worry point with this method is step 2, which involves cutting the cable at the counterweight. The robotics that carries out this step will have to be designed very robustly to guarantee that the counterweight does not accidentally “let go” of the cut tether that connects it to Earth.

Numerous variants of the breeder elevator can be considered. The one that is presented here was chosen mainly for its simplicity.

## 2.4 Redeploy and Splice Buildup

We have just seen that it is possible to use material spooled up at the counterweight to build a new elevator. Better yet, it is possible to use that material to further build up an existing elevator. This leads to a very fast build up method. As we shall see in Section 3, this method remains competitive even for much weaker tether materials than those in [5]. This is in sharp contrast with climber buildup methods that suffer significantly if the tether strength is reduced.

In this method, tether material that gets spooled up at the counterweight is redeployed and spliced to the existing tether, resulting in a tether nearly twice as wide as before. Buildup proceeds as follows:

1. Reel up enough tether material to reach from anchor to counterweight and to provide additional mass to counterweight.
2. Cut the tether at counterweight, and attach it to a new spool.
3. Reel up enough tether material to reach from anchor to counterweight.
4. Attach the end of the tether on the old spool to the tether connected to Earth.
5. Pull attachment point back down. The counterweight splices the material from the two spools together as they are reeled out.

The same remarks apply as for the breeder elevator. The main difference is that we now have to splice tethers at the counterweight. Consumables will probably be needed to do the splice operation. If they are

light enough, these consumables can be attached to the tether coming from Earth, and reeled up with it.

## 3 Evaluation

In this section, we shall compare climber based, reel-to-reel and redeploy and splice buildup methods. I have kept the analysis as simple as possible, so these results are not exact. I have tried to be optimistic when evaluating the climber based method, and pessimistic for the exponential tether methods. I have chosen the same velocity  $v = 200$  km/h (climber speed or reeling speed), as the technological limitations on the velocity seem to be the same in all three cases. The results are strongly in favor of the exponential tether results, despite this bias.

This study assumes that the maximum stress is the same for exponential and uniform-stress tethers. Since exponential tethers are under-stressed over most of their length, it may turn out that smaller safety factors can be used for them, which would make them even more attractive.

For each method, buildup proceeds exponentially with a growth rate that we compute. We shall not consider effects that only occur at the beginning or end of buildup as they are negligible for large amounts of buildup. The results are plotted in Figure 4. For ease of interpretation the growth rates  $\Gamma$  have been converted to doubling times  $t_2 = \ln(2)/\Gamma$ .

### 3.1 Climber Based Buildup

For climber based buildup, we take the ratio of mass per unit time going up the elevator to elevator mass, to evaluate the tether growth rate. The elevator mass is minimized when the counterweight is infinitely distant, so we place ourselves in that case. The elevator mass is computed by integrating the analytical expression for the cross-section [5, 3].

It has been noted [5, 7] that the mass rate for climbers is better when frequent light climbers are used, rather than large climbers that take up the whole capacity of the elevator. Let  $d$  be the distance between successive climbers. The payload capacity  $\sigma A(r_e)$  is split between all the climbers that are below GEO. When a climber is just departing from the



anchor, the weight acting on all these climbers is

$$\sum_{n=0}^{\lfloor \frac{r_g - r_e}{d} \rfloor} g(R_e + nd)M_{cl} \quad (8)$$

The climber mass  $M_{cl}$  should be chosen so that this weight equals the payload capacity. The mass going up the elevator per unit time is simply  $M_{cl}v/d$ .

Combining these results, we get a growth rate of

$$\Gamma_{cl} = \frac{\sigma v}{d \sum_{n=0}^{\lfloor \frac{r_g - r_e}{d} \rfloor} g(R_e + nd) \int_{R_e}^{\infty} \rho \frac{A(r)}{A(r_e)} dr} \quad (9)$$

Figure 4 shows the resulting growth rates. I have plotted a number of different climber spacings. Edwards [5] spaces climbers apart by three days. Closer spacings have also been shown, as they greatly improve the growth rate. However, it isn't clear that faster rates will be easy to achieve in practice as they involve more climbers, lighter climbers and more frequent climber launches. The results shown here are a bit better than in [5], the reason for this mismatch is not clear as Edwards does not detail how his numbers were arrived at.

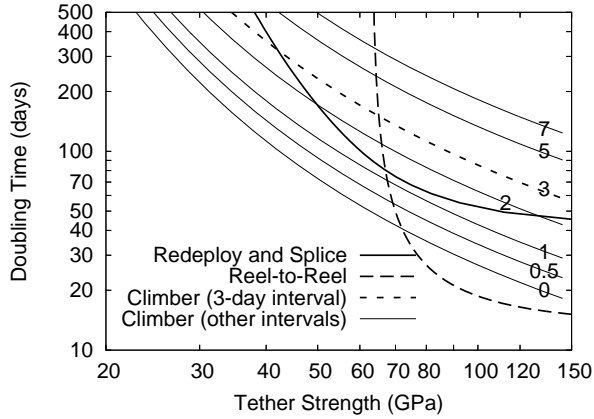


Figure 4: The doubling time for the different buildup methods, as a function of tether strength. Assumes an Earth space elevator, and a tether density of  $1300\text{kg/m}^3$ . The climber method has been evaluated for various climber departure rates indicated on the plot in days.

### 3.2 Reel-to-Reel Buildup

For reel to reel buildup, the growth rate is simply  $-\gamma v$ . From  $\gamma$  we compute the minimum strength that is necessary for reel-to-reel buildup. For simplicity, we consider that the counterweight is located in such a way that it remains in place as the tether is reeled up. This occurs when the mass of the counterweight is  $M_c = -\rho A(r_c)/\gamma$ . This is the mass that an infinitely long extension of the tether, rolled up at the counterweight, would have. Assuming the best case of zero tension at the anchor, we integrate (3) from the anchor until the counterweight boundary condition is satisfied with the desired mass. This condition is  $-\rho A(r_c)g(r_c) = T(r_c)\gamma$ . For taper ratios less than 0.691 this condition is never satisfied, so there is a lower bound to the amount of inverse taper we can make use of. The value of this bound is planet dependent.

Figure 4 shows the results. Reel-to-Reel buildup is the absolute best choice for strengths greater than 72 GPa. At 65 GPa it is nearly identical to climber-based buildup with a three day climber interval. This is remarkable, given how close 65 GPa is to the critical strength of 63 GPa (see Section 1.3).

### 3.3 Redeploy and Splice

For redeploy and splice, we need to determine how far out to place the counterweight. I have chosen to place it at the point where the counterweight mass goes to zero, to avoid having to consider the amount of extra tether to reel for growing the counterweight. For a given taper ratio, this is also the farthest the counterweight can be placed without needlessly increasing the tension at the anchor beyond zero, so it is a conservative assumption.

After one full redeploy and splice cycle, the tether has been grown by a factor  $e^{-\gamma(R_c - R_e)} + 1$ . The full cycle takes a time  $3(R_c - R_e)/v$  for all the reeling operations. In practice, the counterweight would actually move closer to the Earth during the reeling operations, so the factor of three is pessimistic.

The resulting growth rate is

$$\Gamma_{rs} = \frac{v \ln(e^{-\gamma(R_c - R_e)} + 1)}{3(R_c - R_e)} \quad (10)$$

Figure 4 shows the results. Redeploy and splice is always outperformed by the best climber based

methods. However, if climbers can only depart every three days, redeploy and splice is the method of choice from 42 GPa to 67 GPa. I recall Dr. Edwards wondering at the Second Space Elevator conference what could be done if the tether strength dropped significantly below 65 GPa, as the climber method drastically slows down past that point. The redeploy and splice method does as well at 51 GPa as the climber method does at 65 GPa, so it could be the answer.

## Conclusion

In this paper I have introduced exponential space elevators, and studied their basic properties. More importantly, I have shown how these elevators can be used to greatly accelerate the build up phase of space elevator construction. The simple reel-to-reel technique is well suited to strong tethers, while the redeploy and splice method is better suited to weaker tethers. With the reel-to-reel technique, a 51 *GPa* tether can be built up as fast as a 65 *GPa* tether with the currently accepted climber based method. This could mean the difference between feasible and impossible if carbon nanotube materials don't reach expected strengths. At the very least, reducing the buildup time will reduce the amount of time during which the elevator is thin and vulnerable. These new buildup methods also lead to an elevator with a much better ribbon quality, since the final ribbon is entirely pulled up from the ground, rather than being spliced together by climbers at 200 km/h.

Exponential tethers have also been proposed as a faster way to produce a new elevator from an existing one. A breeder elevator could be made that produces a new elevator every few months. All these applications of exponential tethers rely on the fact that mass can be lifted faster by reeling tether material up on an exponential elevator than by using climbers on a uniform-stress elevator. Uniform-stress elevators are the right solution for lifting arbitrary payloads, but when ultra-strong tether material needs to be lifted, exponential tethers are better. Exponential elevators could have uses beyond elevator deployment, to get construction material to other space projects.

The analysis that is presented here ignores the effects of elasticity, and does not take dynamic effects into account. These will have to be considered to build confidence in the new methods. More effort

also needs to be put into pull-down elevators, and the study of the counterweight motion when exponential elevators are used in real-life scenarios.

Today the materials needed to build the space elevator are not yet available. While we wait, we should work on lowering the minimum strength that is needed for space elevator construction. In this paper I have shown how getting rid of uniform-stress tethers can be a step in that direction. I can only encourage the reader to think of other common assumptions that can be knocked down to bring the space elevator closer to reality.

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## References

- [1] Y. Artsutanov, "V kosmos na electrovoze (into space with the help of an electric locomotive)," *Komsomolskaya Pravda*, 1960.
- [2] V. Lvov, "Sky-hook: Old idea," *Nature*, vol. 158, no. 3803, pp. 946–947, Nov. 1967.
- [3] J. Pearson, "The orbital tower: a spacecraft launcher using the Earth's rotational energy." *Acta Astronautica*, vol. 2, pp. 785–799, 1975.
- [4] A. C. Clarke, *The Fountains of Paradise*. Warner Books, 1978.
- [5] B. Edwards and E. Westling, *The Space Elevator*. Spageo Inc., 2002.
- [6] J. Cline, "Comments on Edwards tether," <http://www.kestsgeo.com/pages/cobetse.html>, (mirrored at Cline's request at <http://www.gassend.com/spaceelevator/cline/>), 2002.
- [7] E. A. Westling, "Space elevator budget and economics," Sept. 2003, talk at the 2003 Space Elevator Conference.