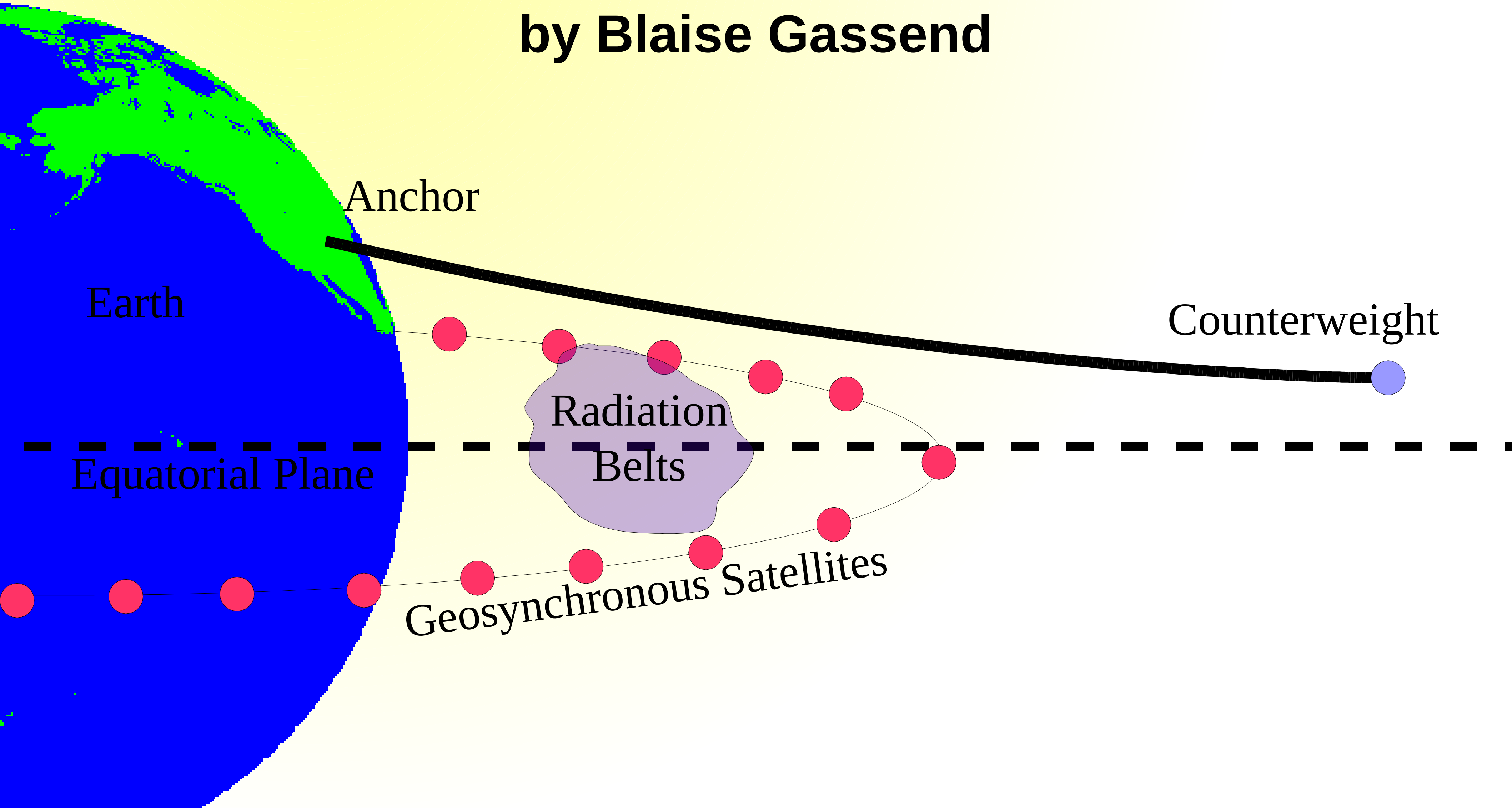
Non-Equatorial Uniform-Stress Space Elevators

by Blaise Gassend



Introduction

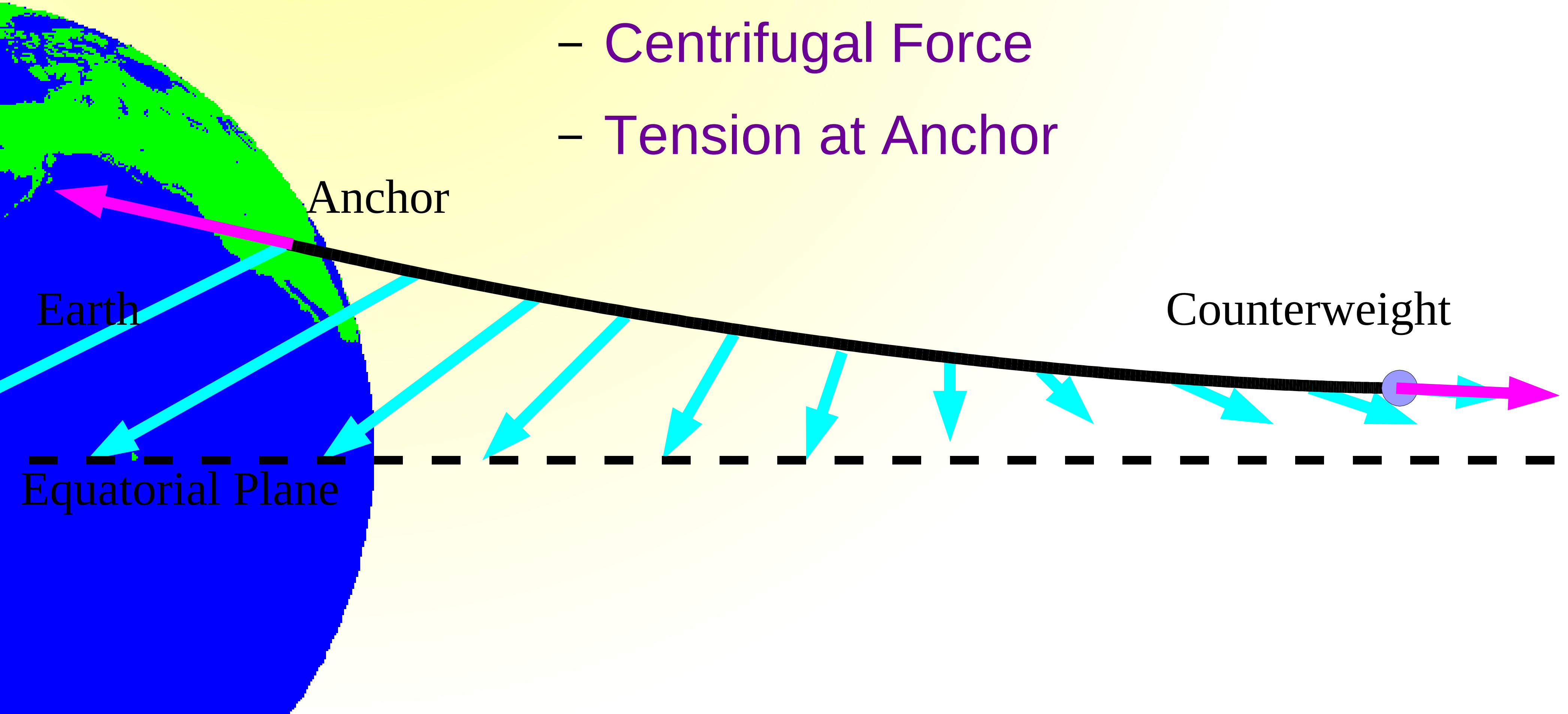
- Reasons for non-equatorial space elevators:
  - Added flexibility for anchor location.
  - Avoid geosynchronous orbit.
  - Avoid most intense part of radiation belts.
  - Avoid Martian moons.
- Questions on non-equatorial space elevators:
  - Maximum latitude.
  - Effect on payload to elevator mass ratio.

Outline

- Static Equations
- Properties of Non-Equatorial Elevators
  - Description of Solutions
  - Reachable Lattitudes
- Practical Concerns
  - Payload to Elevator Mass Ratio
  - Horizontal Force on Anchor
  - Deployment

Equilibrium of Elevator

- Forces acting on Elevator
  - Gravity
  - Centrifugal Force
  - Tension at Anchor



Basic Equations

- No shear stress: $\dfrac{d\vec{r}}{ds} = \dfrac{\vec{T}}{T}$
- Newton's second law: $\dfrac{d\vec{T}}{ds} = \rho A \vec{\nabla} V$
- Uniform-stress condition: $T = \sigma_0 A$
- Counterweight boundary: $\vec{T} = -m \vec{\nabla} V$

Basic Equations

- No shear stress: $\dfrac{d\vec{r}}{ds} = \dfrac{\vec{T}}{T} = \vec{u}$
- Newton's second law: $\dfrac{d\vec{T}}{ds} = \dfrac{\rho}{\sigma_0} T \vec{\nabla} V$
- Uniform-stress condition: $T = \sigma_0 A$
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Basic Equations

- No shear stress: $\frac{d\vec{r}}{ds} = \vec{u}$ $\int \frac{1}{T}\frac{dT}{ds} = \frac{\rho}{\sigma_0}\cancel{T}\,\vec{\nabla} V \cdot \vec{u}$
- Newton's second law: $\cancel{T}\frac{d\vec{u}}{ds} = \frac{\rho}{\sigma_0}\cancel{T}\,(\vec{\nabla} V)_\perp$
- Uniform-stress condition: $T = \sigma_0 A$
- Counterweight boundary: $\vec{T} = -m\vec{\nabla} V$

Basic Equations

- No shear stress: $\frac{d\vec{r}}{ds} = \vec{u}$

$$T = T_0 e^{\frac{\rho}{\sigma_0} V}$$

- Newton's second law: $\frac{d\vec{u}}{ds} = \frac{\rho}{\sigma_0}(\vec{\nabla} V)_\perp$
- Uniform-stress condition: $T = \sigma_0 A$
- Counterweight boundary: $\vec{T} = -m \vec{\nabla} V$

Basic Equations

- Taper Equation $A = A_0 e^{\frac{\rho}{\sigma_0} V}$
- Shape Equation $\frac{d^2 \vec{r}}{d s^2} = \frac{\rho}{\sigma_0} (\vec{\nabla} V)_\perp$
- Counterweight boundary: $\vec{T} = -m \vec{\nabla} V$

$$A = A_0 e^{\frac{\rho}{\sigma_0} V}$$

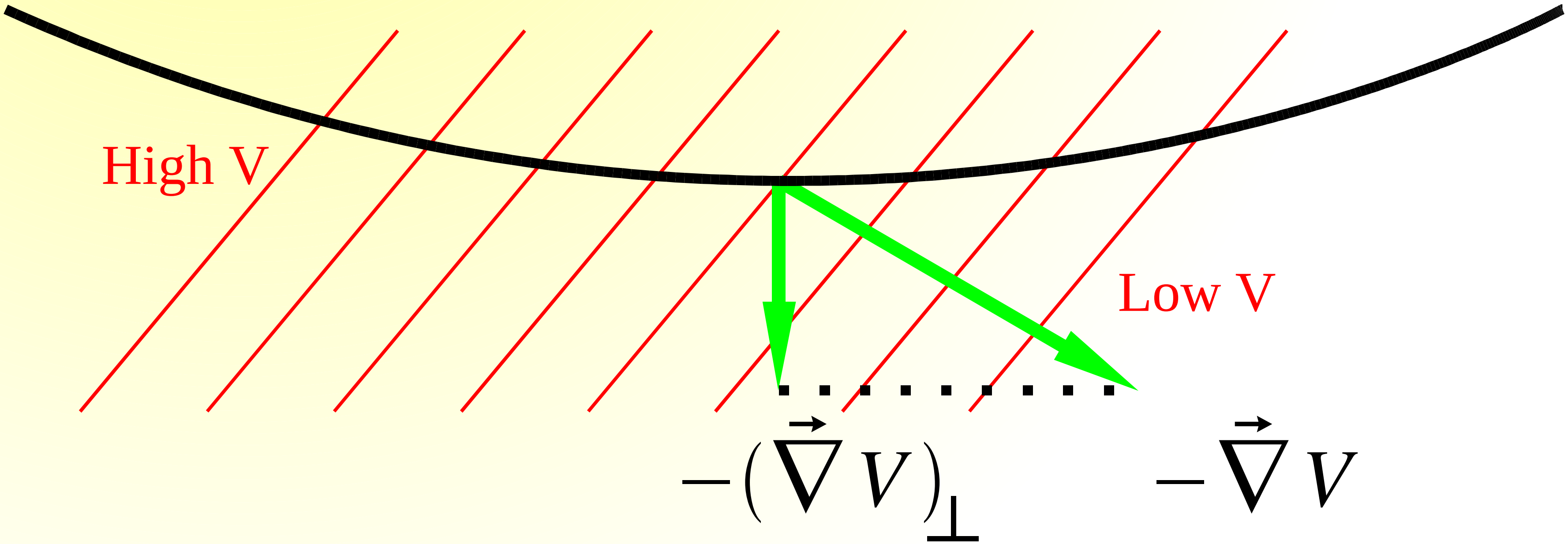
Taper

- Taper Equation can be integrated without knowing the potential.
- Taper ratio increases exponentially with potential barrier.
- Compare Strength to Weight ratio with potential barrier to determine if elevator is easy to make.
  - Ratio is 0.97 for Edwards elevator.

$$\frac{d^2\vec{r}}{ds^2} = \frac{\rho}{\sigma_0}(\vec{\nabla} V)_\perp$$

Shape

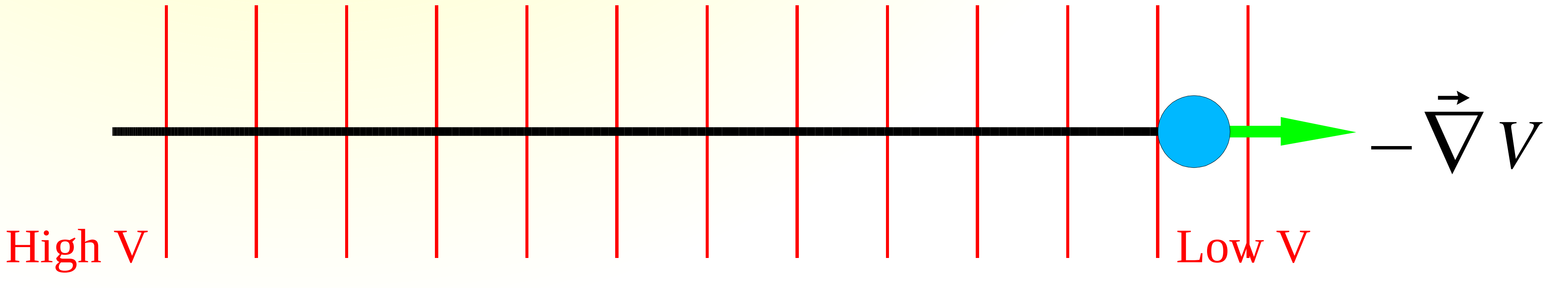
- Tether curves towards areas of higher potential.



Counterweight Boundary Condition

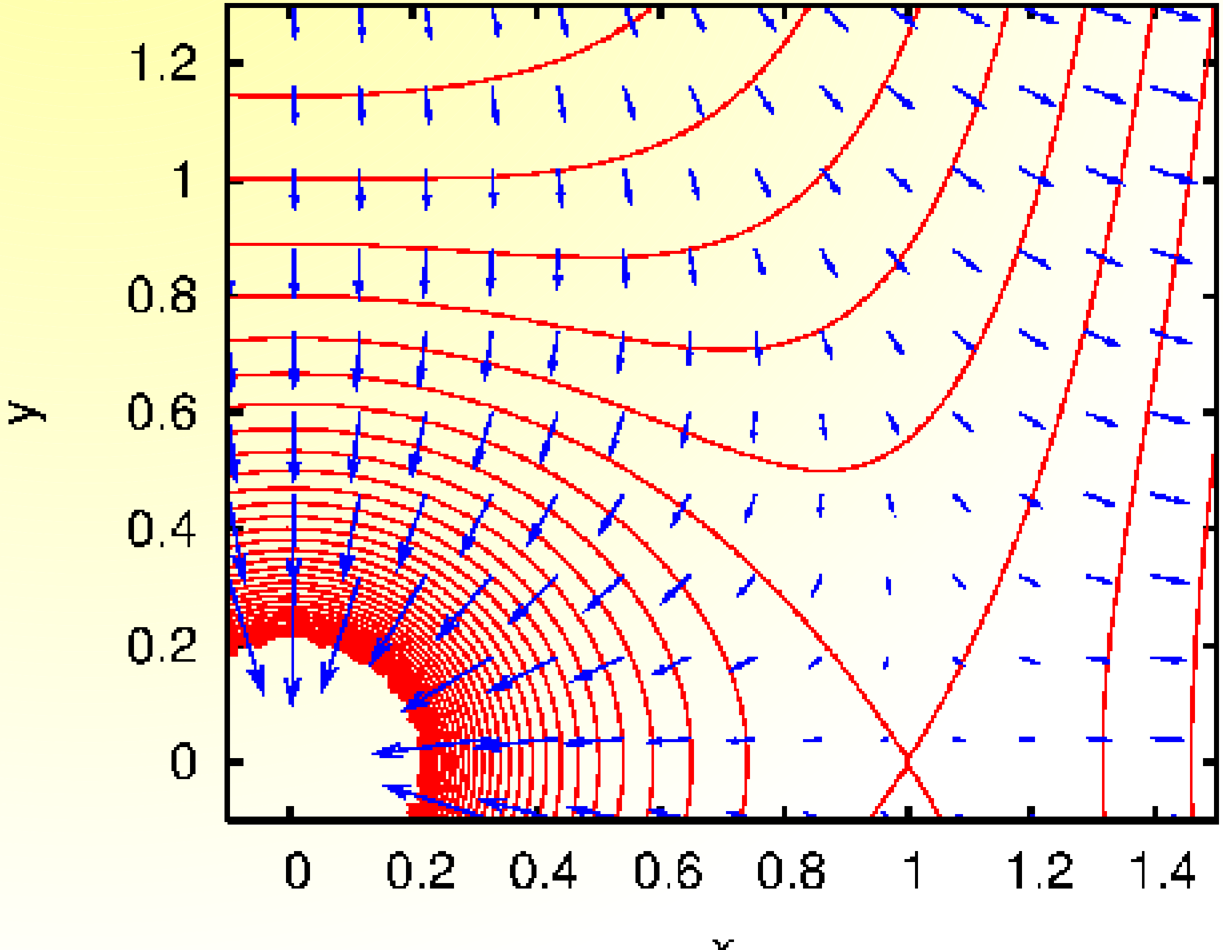
$$\vec{T} = -m\vec{\nabla} V$$

- The counterweight boundary condition determines where we can terminate the elevator:
  - Tension must be parallel to local gravity field
  - Gravity field must point away from the end of the tether.
  - Mass of the counterweight must be just right.



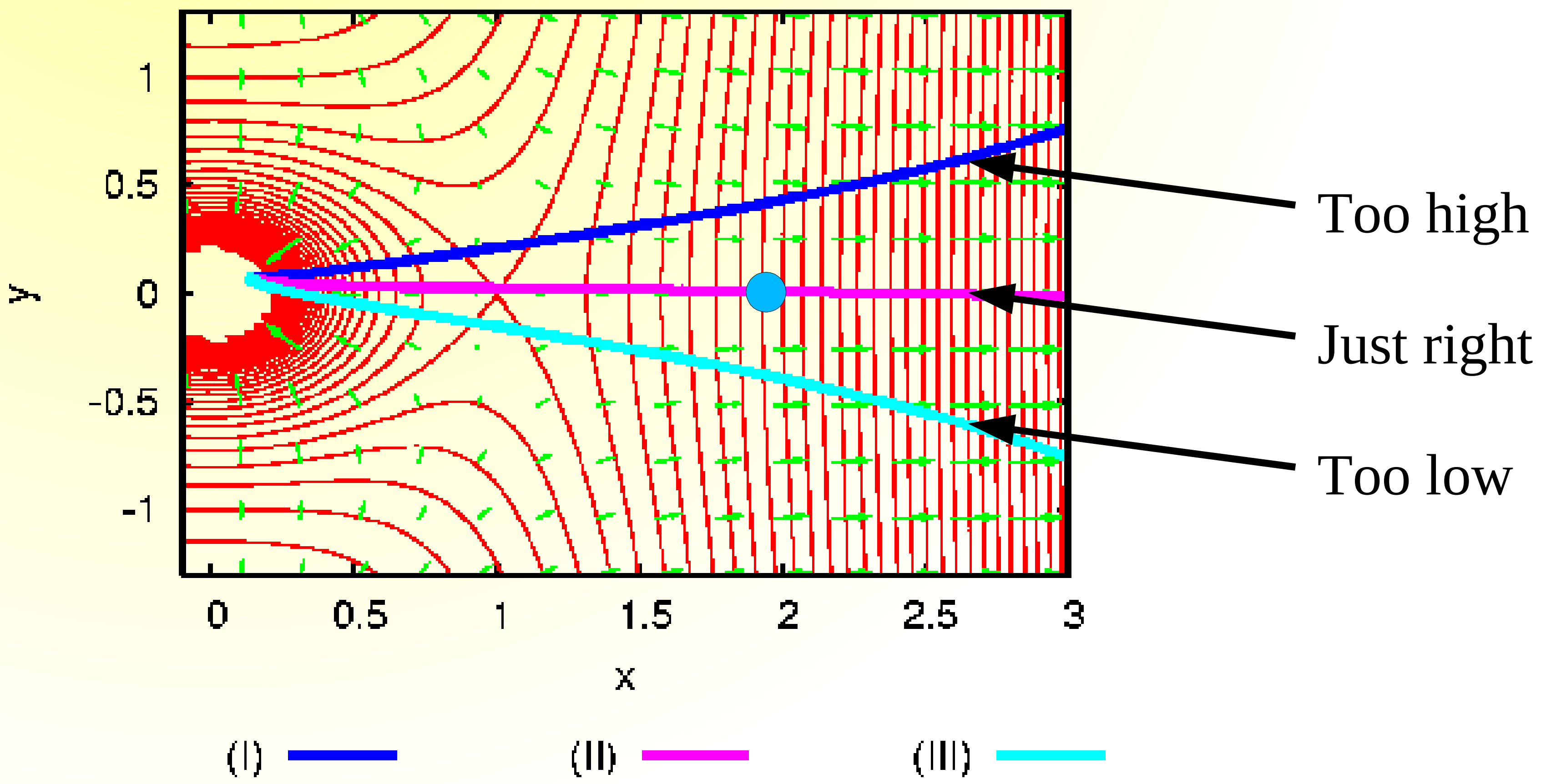
The Rotating Coulomb Potential

$$V = -V_0\left(\frac{1}{2}\hat{r}^2 + \frac{1}{\hat{r}}\right) \qquad \vec{g}(\vec{\hat{r}}) = g_0\left(\vec{r}_\perp - \frac{\vec{r}}{\hat{r}^3}\right)$$



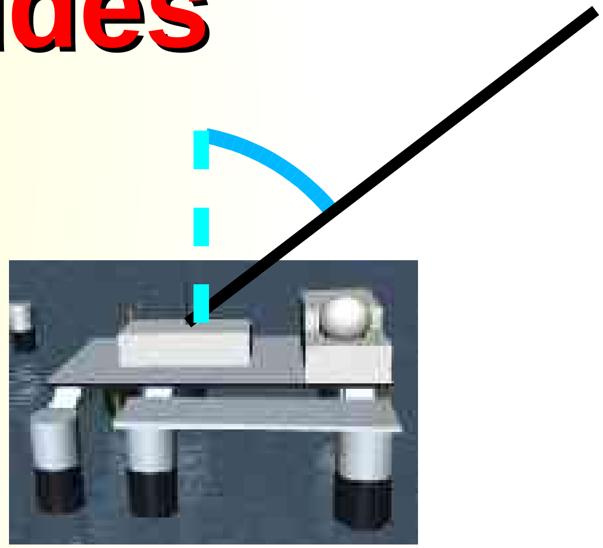
Solutions in the Coulomb Potential

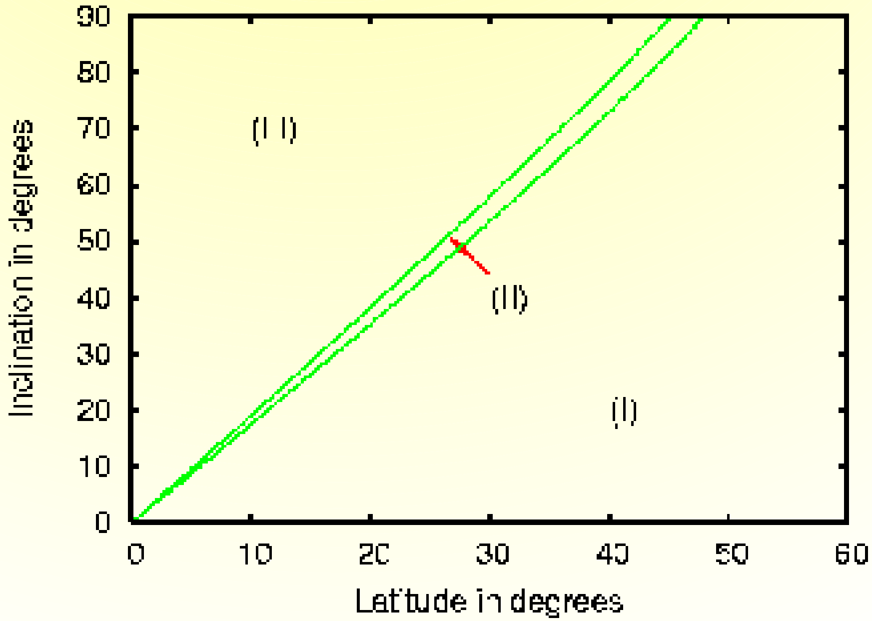
- Solutions are planar.
- Boundary not always satisfied.

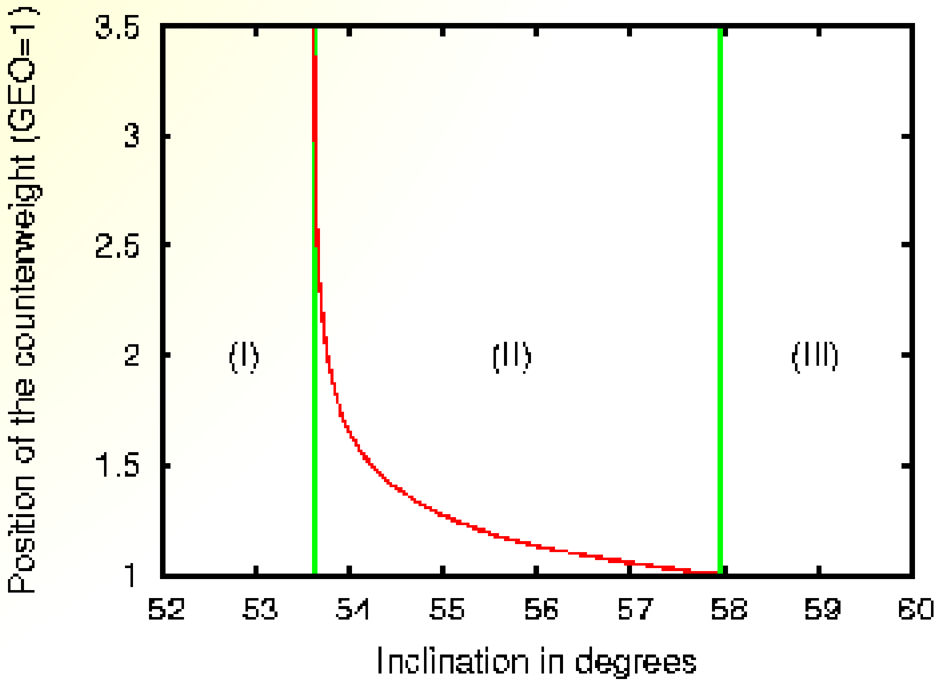


Reachable Latitudes

- For each latitude there is a small range of tether inclinations





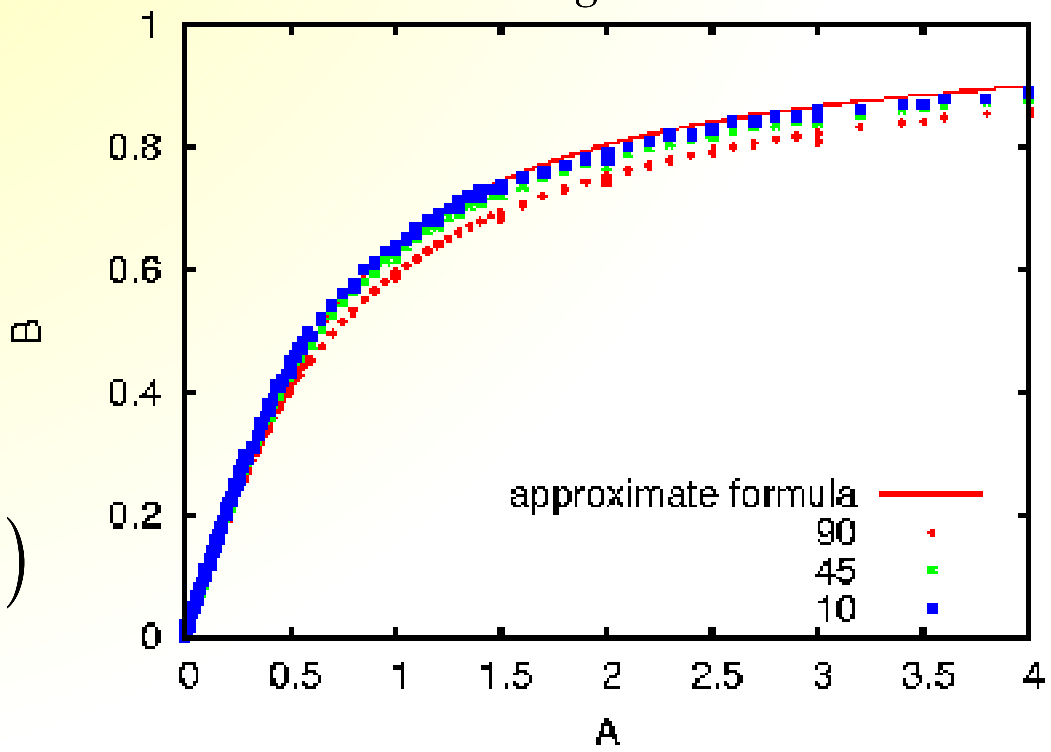


The Arc-Tangent Formula

- Empirical formula works well when synchronous altitude far above surface.

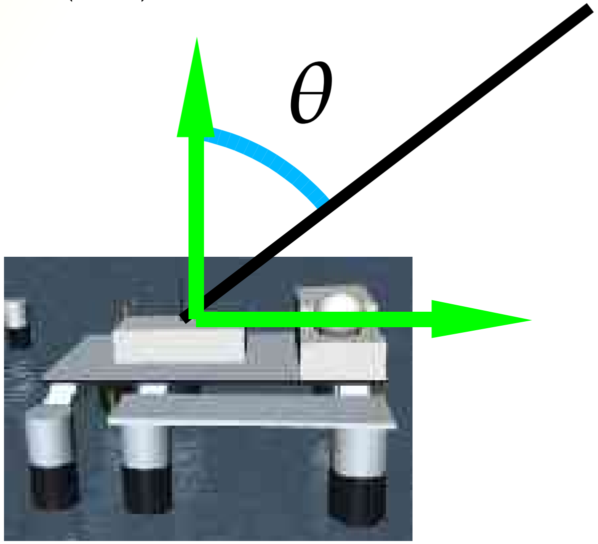
$$\frac{\pi}{2}\frac{Latitude}{Inclination} = \arctan\left(\frac{\pi}{2}\frac{r_e \sigma_0}{r_g \rho V_0}\right)$$

$$\frac{\pi}{2} A = \arctan\left(\frac{\pi}{2} B\right)$$

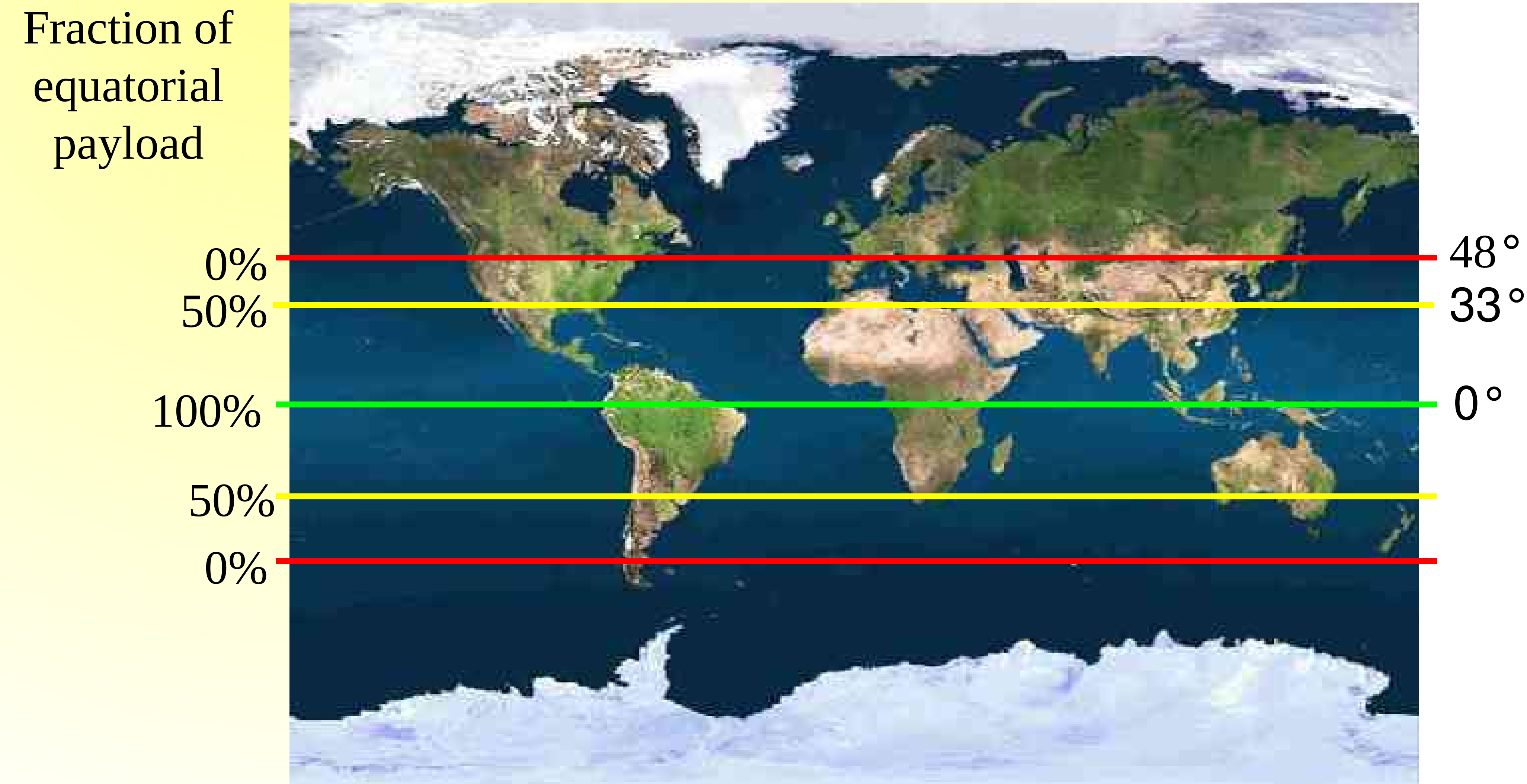


Inclination at Base of Elevator

- Payload (vertical component)
  - Taper ratio roughly independent of latitude.
  - Elevator length roughly independent of latitude.
  - Inclination of anchor depends on latitude.
  - Payload to mass ratio goes like $\cos(\theta)$
  - Can actually get past this.
- Horizontal force
  - Anchor needs to provide continuous thrust.



How far can we go?



Deployment

- Deployment of initial tether needs to be equatorial.
- Can move away from equator during buildup phase.
- May have to change longitude if target longitude occupied by geosynchronous satellite.

Conclusion

- Have covered:
  - Statics of non-equatorial space elevators
  - Where they can be
  - How much they can lift
- Needs more study:
  - Dynamic effects
  - Changes due to climber presence
  - More effort on solving shape in limit cases
- Brad, what question do <u>you</u> want answered?

THE END

More details in the paper!

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Do you have funding for Space Elevator research?