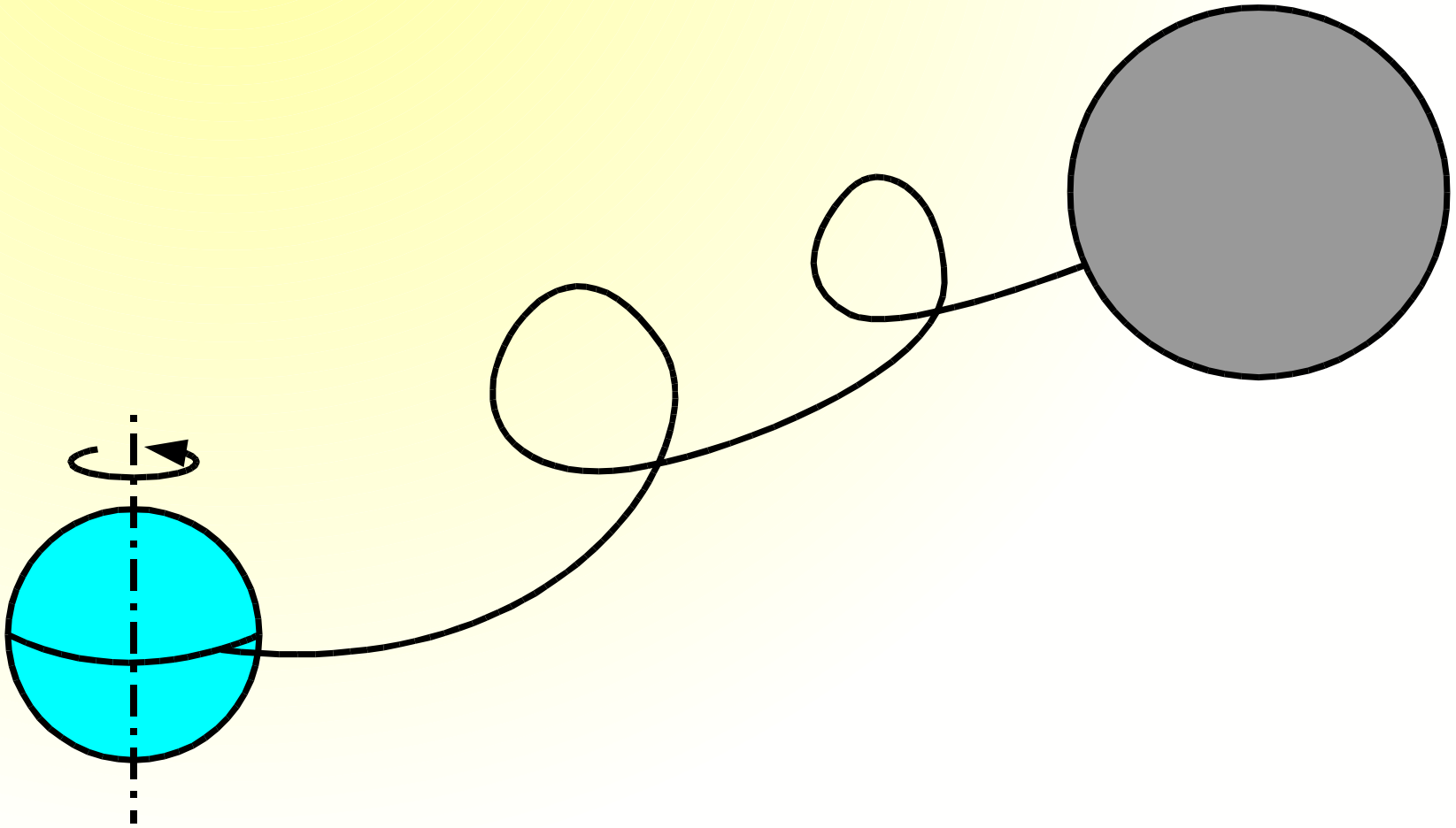
Linear Dynamics of the Space Elevator in the Absence of Climbers

by Blaise Gassend



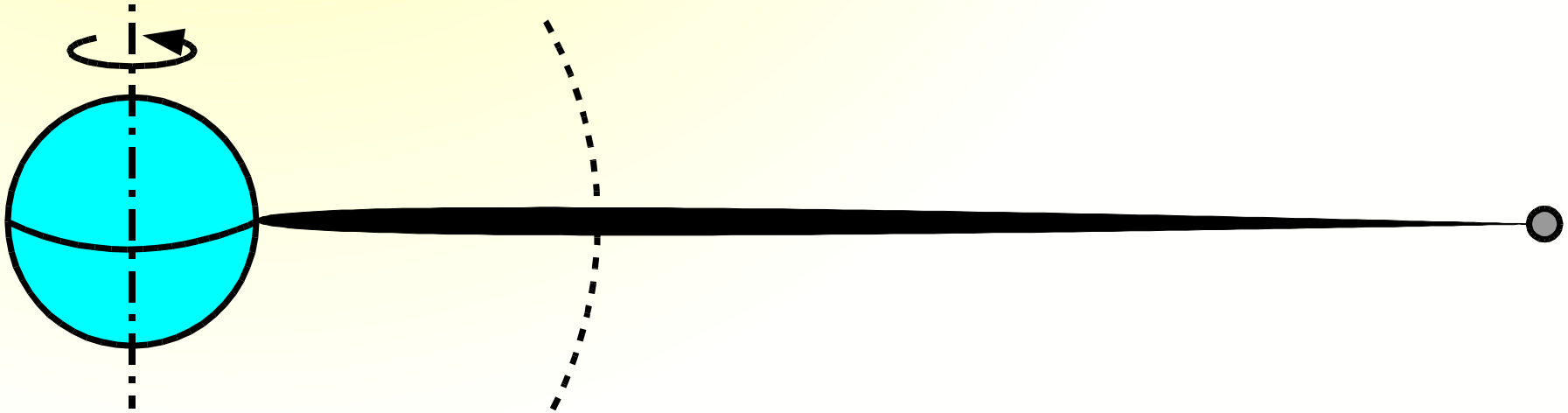
Forces

Idealized System

- Gravity
- Centrifugal Force
- Cable Tension
- Coriolis Force

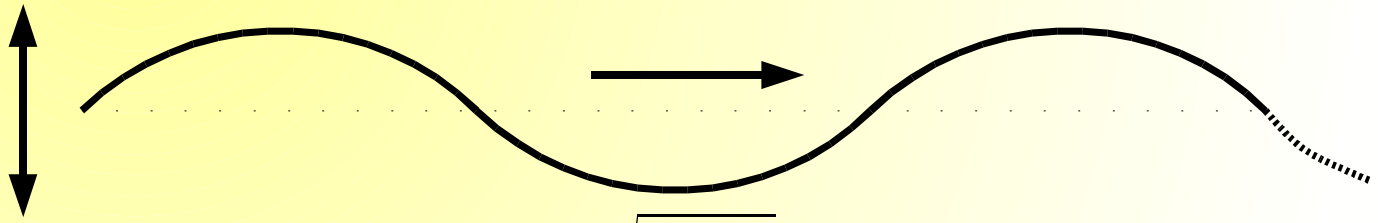
Perturbations

- Lunar Tidal Forces
- Solar Tidal Forces
- Fluttering in Wind
- Climbers



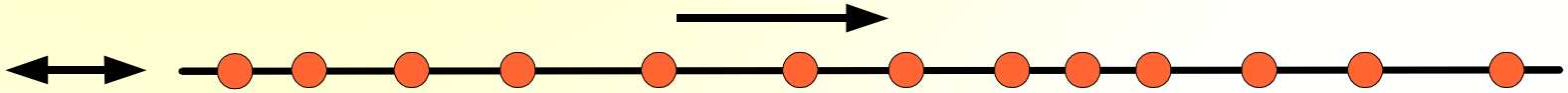
Waves on a Uniform Cable

Transverse motion (Side-to-Side)



- Velocity $c = \sqrt{\frac{\sigma}{\rho}} \approx \sqrt{\frac{65 \cdot 10^9}{1300}} \approx 7 \text{ km/s}$
- Impedance $f = c\,\rho\,A \approx 7 \cdot 10^3 \cdot 1300 \cdot 10^{-6} \approx 9 \text{ N/(m/s)}$

Longitudinal motion (Up-Down)



- Velocity $c = \sqrt{\frac{E}{\rho}} \approx \sqrt{\frac{10^{12}}{1300}} \approx 27 \text{ km/s}$
- Impedance $f = c\,\rho\,A \approx 27 \cdot 10^3 \cdot 1300 \cdot 10^{-6} \approx 35 \text{ N/(m/s)}$

(Impedance suggests damping method and maintaining tension at anchor)

Uniform Cable with Fixed Ends

Resonant frequencies occur when wavelength is a multiple of two times the length of the cable.

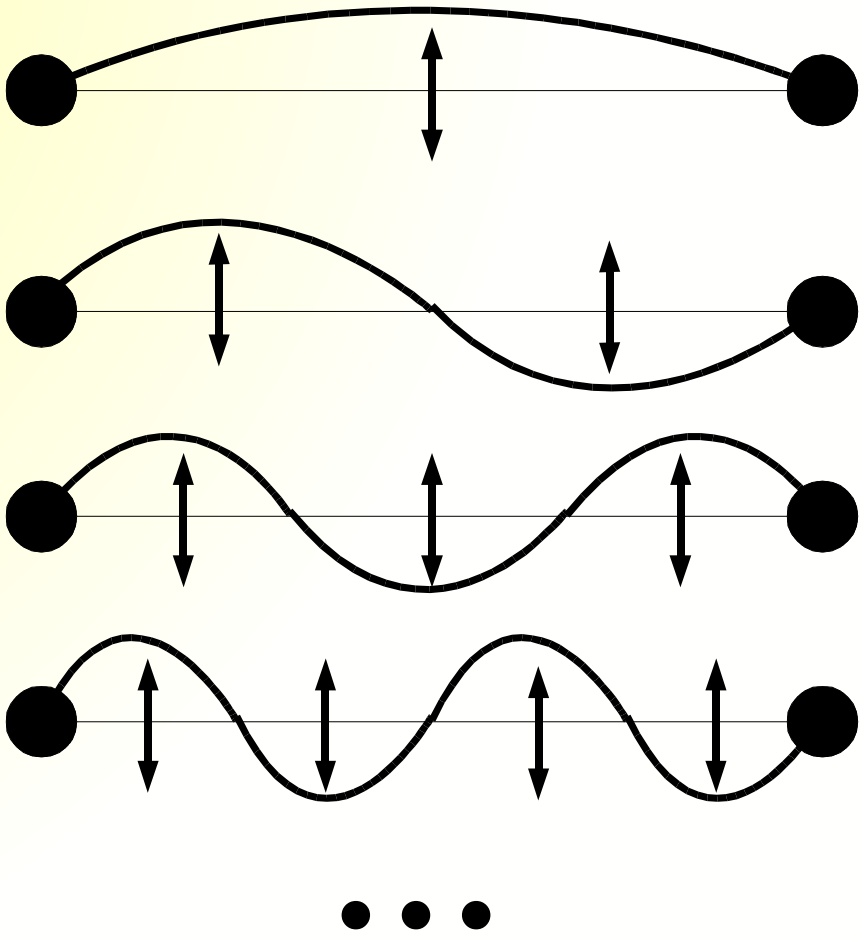
Assuming 91 000 km cable:

Longitudinal (Up-Down)

- 1h51, 56min, 37min, 28min, 22min, ...

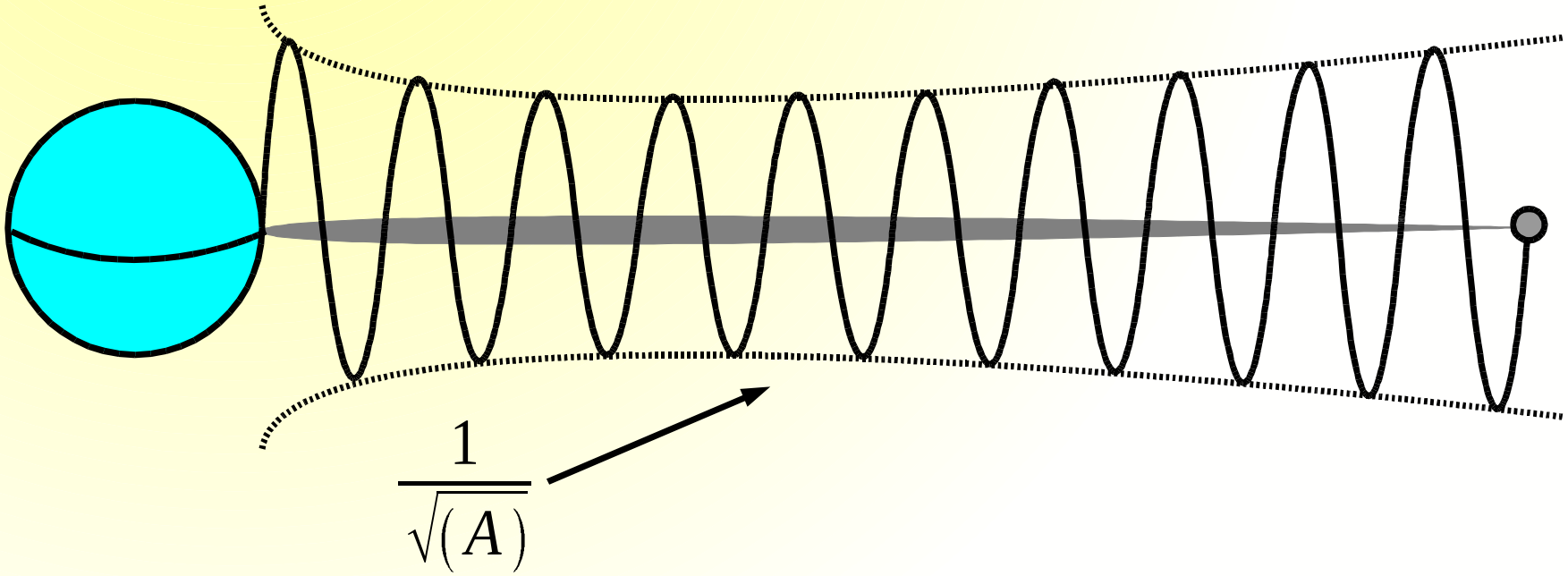
Transverse (Side-to-Side)

- 7h13, 3h37, 2h24, 1h48, 1h26, ...



Effect of Cable Tapering

For slow tapering, the envelope of the modes is changed because more energy is needed to make the cable oscillate where it is wide.

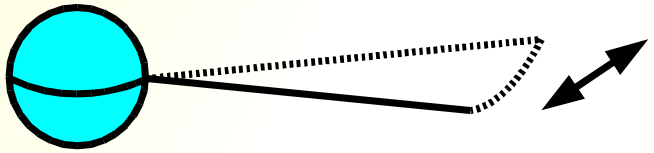


(Impacts debris avoidance and amplitude needed to break cable.)

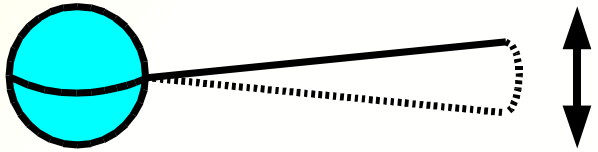
Pendular Modes

- Only one extremity of our cable is really fixed, but at low frequencies, the counterweight can move.
- Hardly affects the modes we have seen, but allows "pendular" modes.

East-West $T \approx 5\,\text{days}$



North-South $T \approx 1\,\text{day}$



Up-Down $T \approx \sqrt{\frac{\sigma}{E}}\,\text{days} \approx 6\,\text{h}$

The 24 Hour Mode

- Solar and Lunar perturbations near 12h in equatorial plane, 24h out of plane.
- Out of plane pendular mode inherently has a period near 24h.
- If cable stiffness or stress decreases, 12h modes may be seen in the equatorial plane.

Lunar Perturbation

- Causes oscillations with amplitude on the order of 25km, in the absence of active compensation.

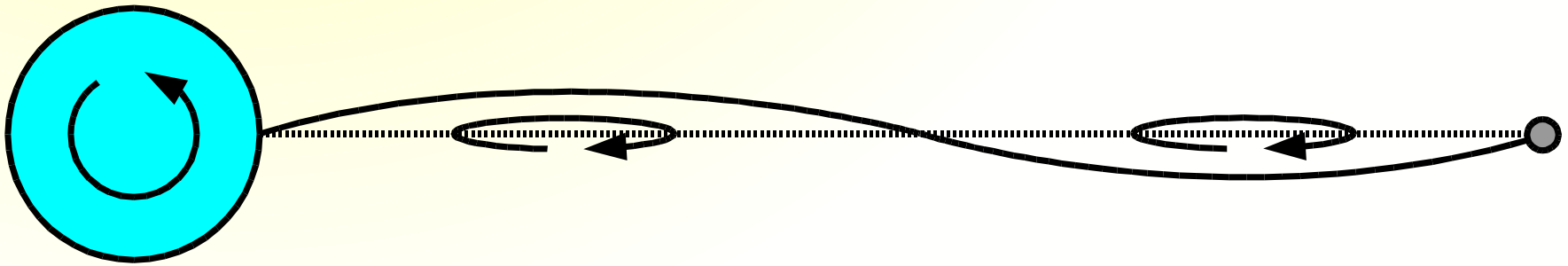
Solar Perturbation

- About 2.5 times weaker than Moon.

The Coriolis Effect

The coriolis effect causes coupling between the up-down and east-west modes.

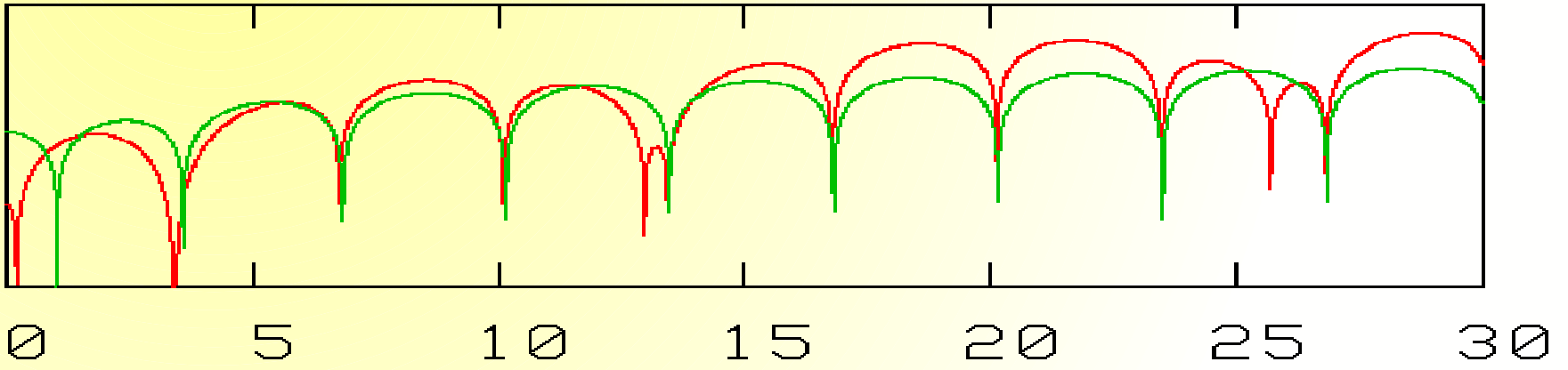
- The coupling is weak because the frequencies in the two directions do not match well.
- Only the lowest modes undergo a noticeable shift.
- Coupling may be exploited to cause small orthoradial motion of counter-weight faster than transverse wave velocity would allow.



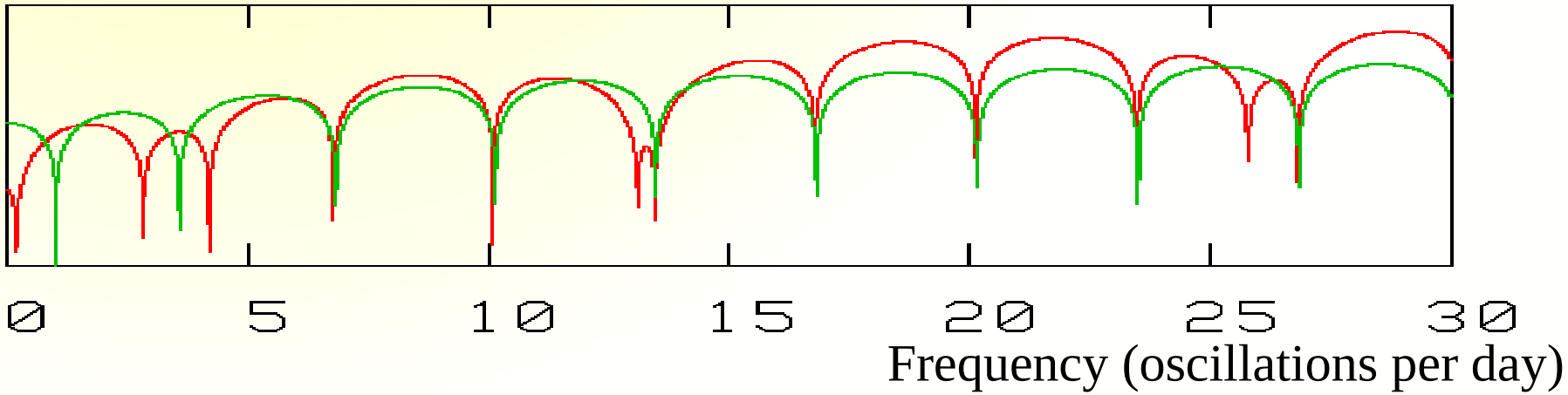
The Coriolis Effect

Low peaks correspond to resonances.

Without Coriolis Effect



With Coriolis Effect



Conclusion

- The dynamics seem pretty tame.

We have covered:

- Forces involved in Space Elevator dynamics.
- Oscillation modes of the elevator.
- A mode exists near 24h but the resonance with Solar and Lunar tidal forces is low enough to be of no concern.
- A few ideas on control using anchor motion.

Needs to be studied:

- Non-linear effects, twisting motion.
- Effect of climbers on modes.
- Deployment phase, cable breaks.

THE END

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Effect of Tapering (Equations)

Not tapered

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Tapered

$$\frac{\partial^2 u}{\partial t^2} = \frac{c^2}{A} \frac{\partial}{\partial x}\left(A \frac{\partial u}{\partial x} \right)$$