

Implicit and Explicit Techniques for Space Elevator Simulation

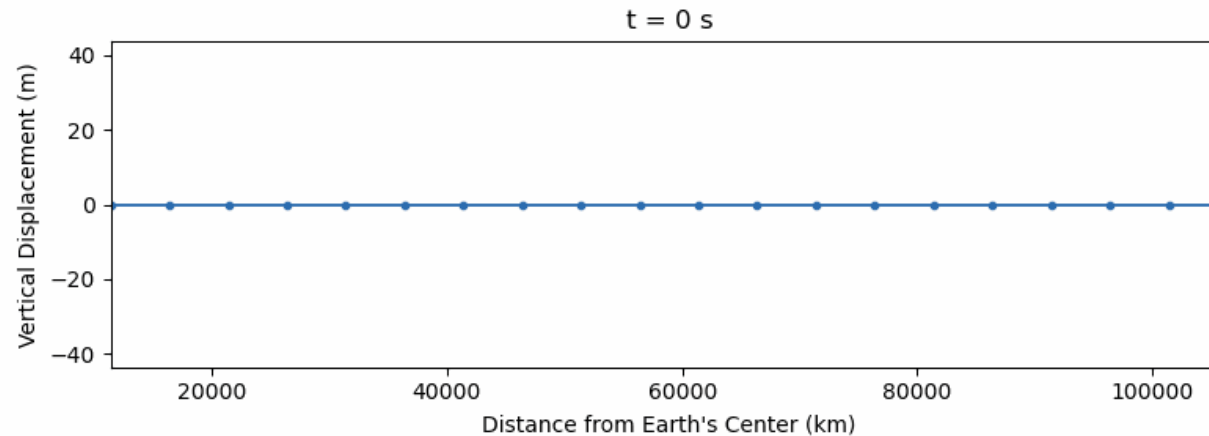
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<https://gassend.com/spaceelevator/isec2026>

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Simulating a Tether Is Easy!



- When you set out to simulate a space elevator, the most natural approach is to represent the elevator using a mass and spring model.
- Within an hour -- 5 minutes with AI -- you can be looking with satisfaction at a nice simulation of the space elevator's tether.
- Note: 1-D simulation with only up-down motion, integrated using Forward Euler.

Adding the Climber

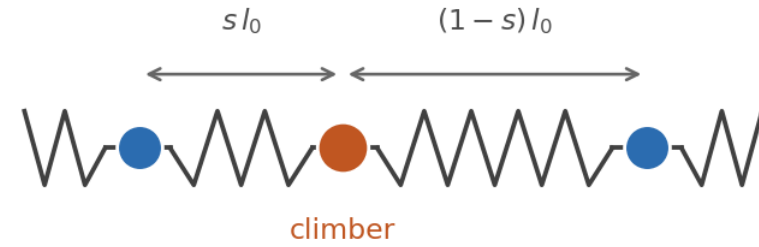
Before: plain tether segment



k, l_0

insert
climber

After: climber splits the segment

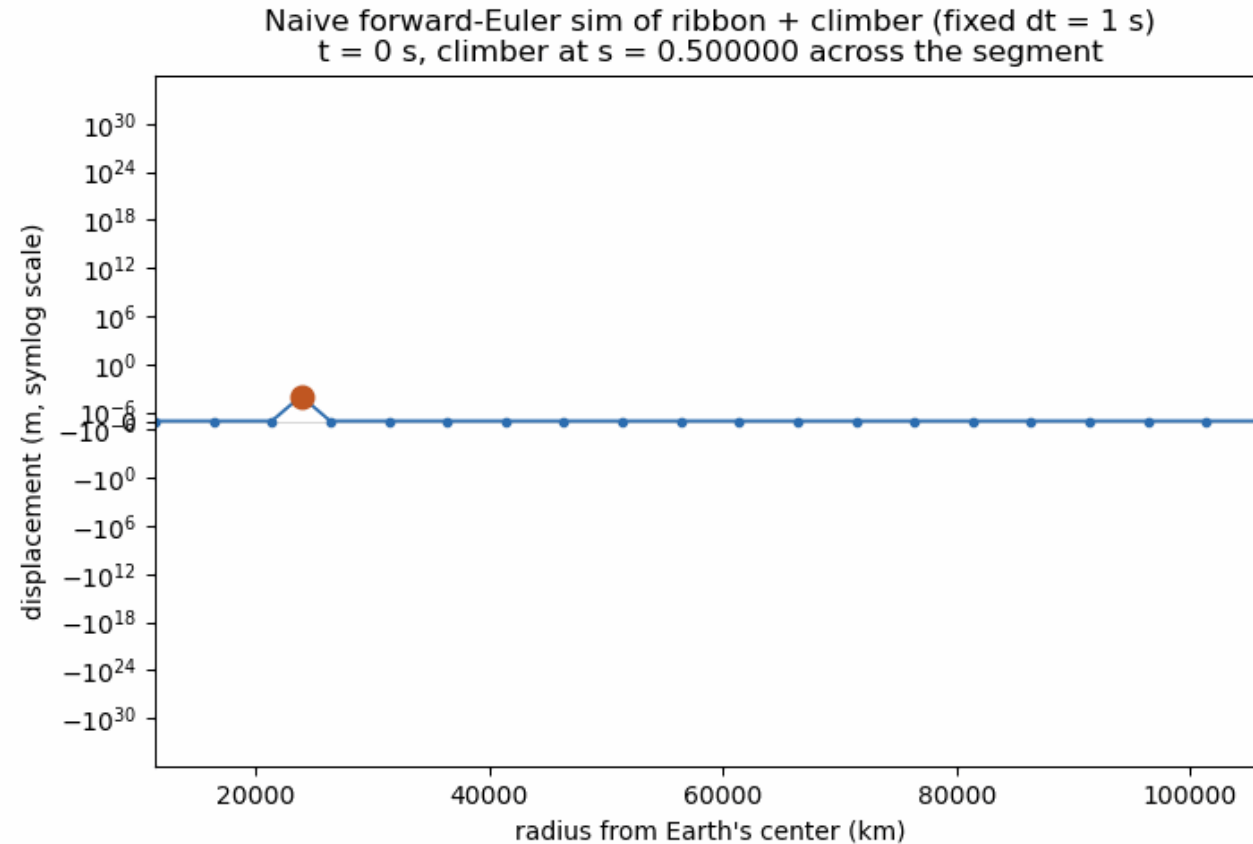


$k/s, s l_0$

$k/(1-s), (1-s) l_0$

To add a climber, the natural approach is to replace the spring that the climber is located on with two springs whose stiffness and length depend on how far along the climber is.

Climbers Make Things Harder!



Unfortunately, the simulation now blows up.

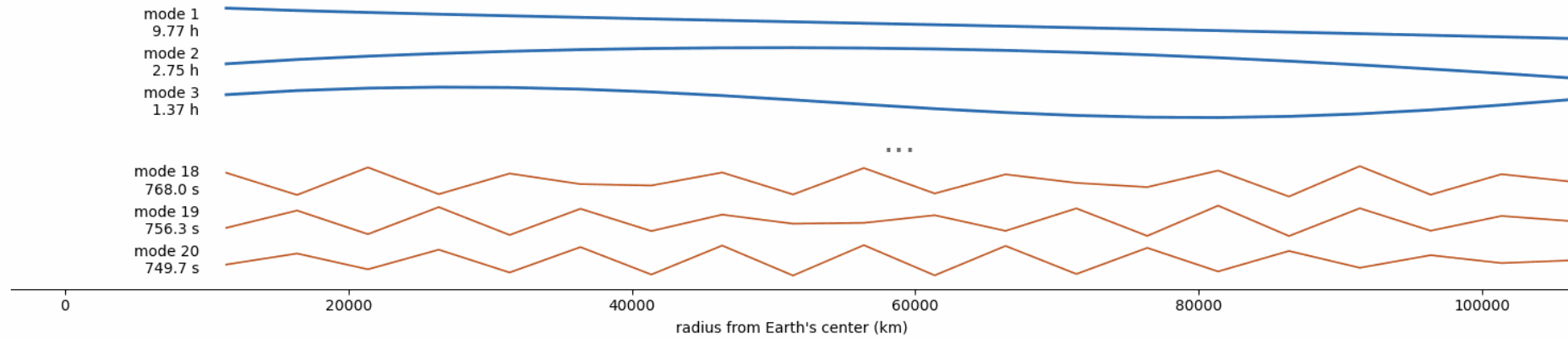
About This Presentation

- These slides were **heavily produced with AI assistance** (Claude).
- The underlying ideas have been **thoroughly vetted by the author**.
- The figures are **illustrative** and may contain inaccuracies -- the code behind them was not checked in detail.
- Many simulations shown are **1-D**, modeling only the vertical (longitudinal) direction, for simplicity.
- The integrators shown mostly use basic **explicit/implicit Euler** for simplicity of exposition -- the same stability effects occur with higher-order methods (RK4, etc.); Euler just makes the story easiest to see.
- The underlying numerical-methods material itself isn't very novel -- it's standard stiff-ODE theory. What's novel here is applying it explicitly to space-elevator tether and climber dynamics. The goal is to help the SE simulation community understand and get past this common pain point.

Coming Up

1. Why the blowup happens
2. What have people been doing about it
3. Implicit methods
4. Why the blowup really happens
5. IMEX method

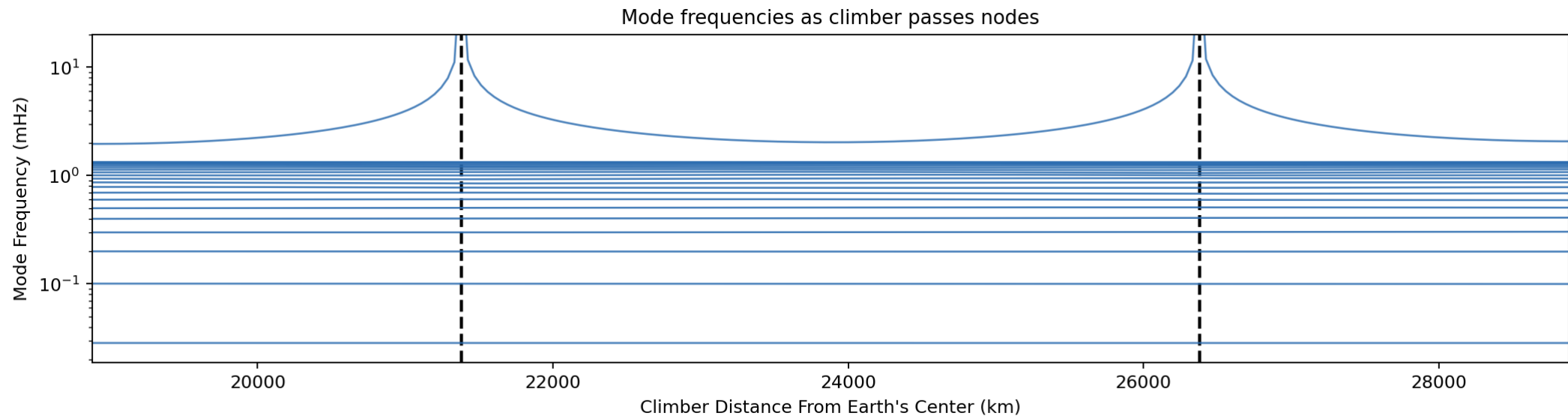
System Modes



- A system near equilibrium can be linearized.
- The linearized system is equivalent to a collection of independent harmonic oscillators, possibly damped.
- The low frequency modes are typically a good representation of the real system.
- The high frequency modes usually are not a faithful representation.

Mode Shift As Climber Moves

- As the climber approaches a node ($s \rightarrow 0$), the short spring stiffness k/s diverges.
- As a result, the highest frequency mode approaches infinity.
- The system becomes **stiff**.

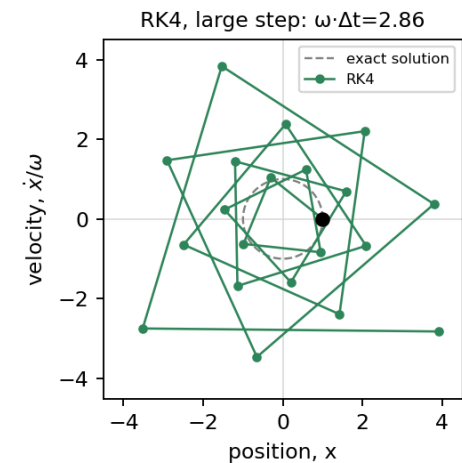
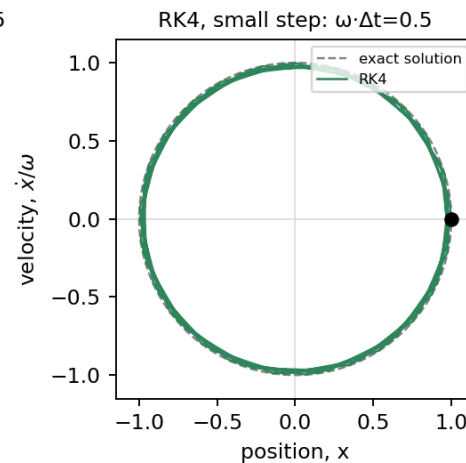
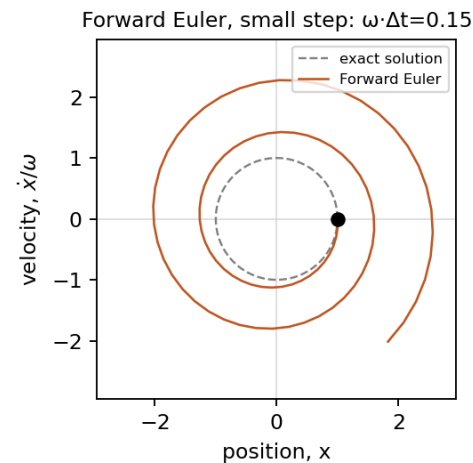
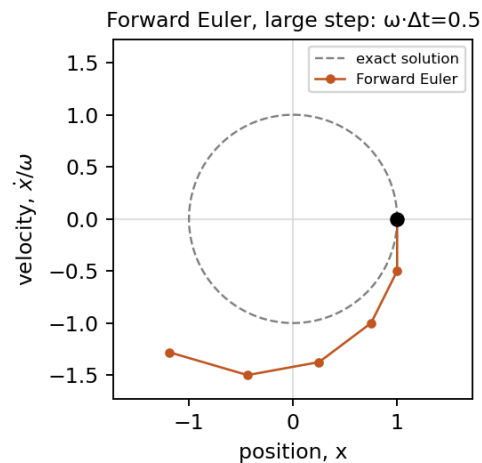


Explicit Methods Divergence

- The simplest simulation methods to implement are called **explicit**.
- You use derivatives from **past** times to compute state variables at **future** times.

Forward Euler: $f(t + \Delta t) = f(t) + \dot{f}(f(t)) \cdot \Delta t$

- Explicit methods diverge for Δt too large compared with the frequency of the mode being simulated.

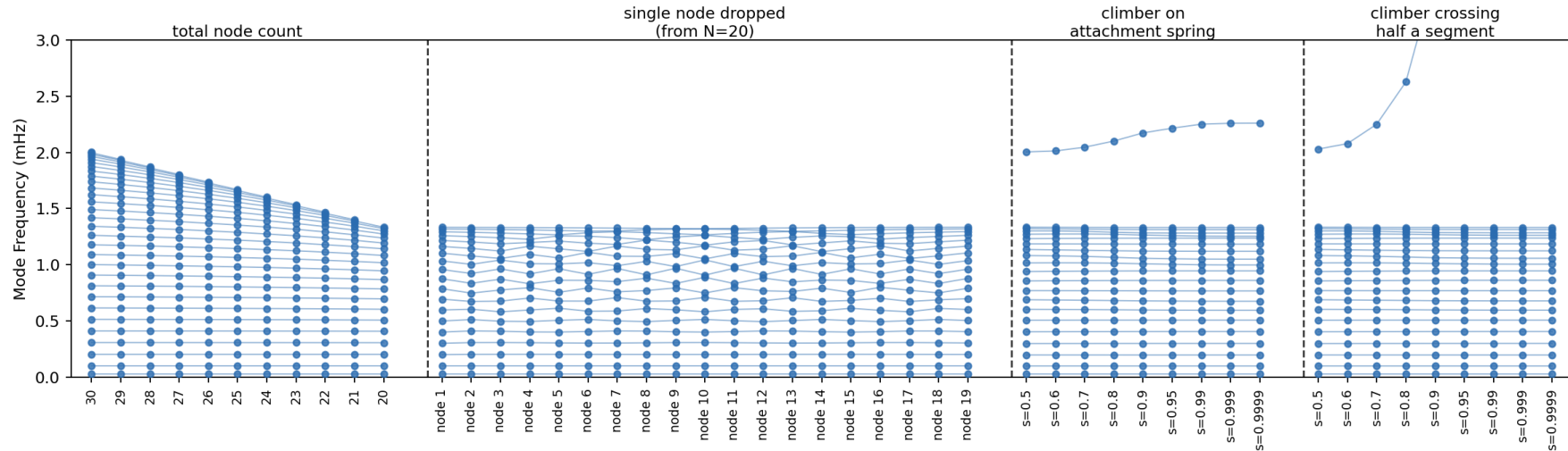


- **Mystery solved!** The naive climber simulation diverges because the highest frequency diverges.

How People Have Been Taming the Divergence

- **Add/drop nodes:** as climber approaches node, drop node, then reinsert as the climber moves away.
 - Unclear where to put the mass of the dropped node. Can't get right to the surface of the Earth.
 - Dropping a node changes DOF count, losing mode energy -- not a problem. But also distorts modes.
- **Resample the tether:** re-discretize node placement as the climber moves to avoid closely approaching node.
 - Inter-node spacing more uniform. But complex. Still need add/drop or you get large variation in node spacing with time (unnecessarily small Δt). Still can't deal with climber right at Earth surface.
 - Resampling damps the high frequency mode. Not a problem.
- **Connect the climber to its attachment point via a spring:** avoids infinite stiffness when passing tether nodes.
 - Sudden attachment point velocity change introduces spurious impulses in the tether.
 - Climber can oscillate relative to attachment point. High frequency, not a problem.

Effect of Modifications on Modes



- Low frequencies remain unchanged when resampling with different numbers of nodes (left) or an extra spring (center-right). Adding/dropping a node (center-left) is more disruptive to the mode frequencies. Unmodified model is shown for comparison (right). Mode 1 stays within 0.02841-0.02844 mHz throughout.
- Modifications have limited impact on accuracy, as long as there are enough nodes to push the impacted modes to irrelevant high frequencies. Resampling and climber attachment via a spring outperforms adding/dropping nodes.

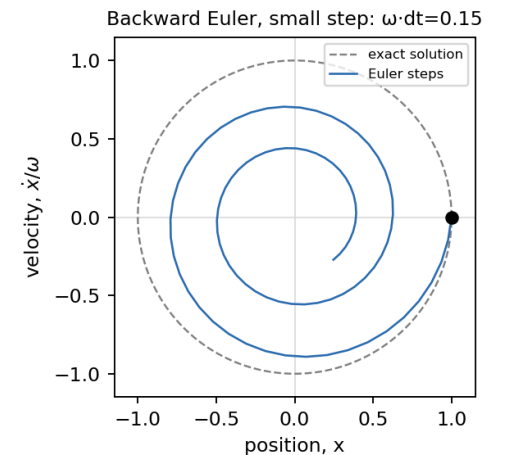
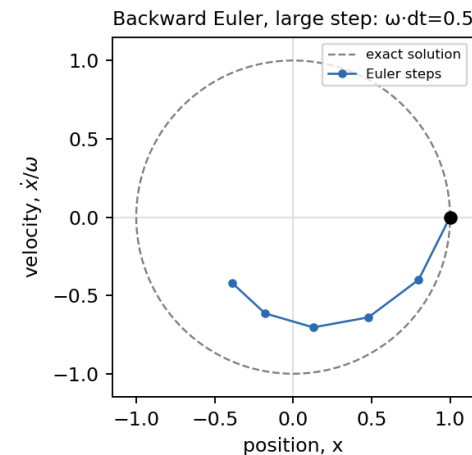
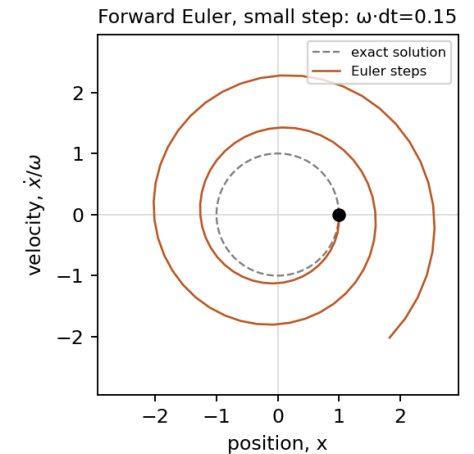
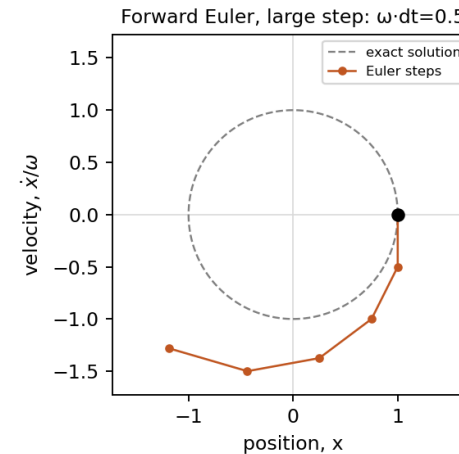
Implicit Integration: Pay per Step, Choose Your Bandwidth

- Implicit methods compute the state variable at **future** times by using derivatives from **future** times.

Backward Euler:

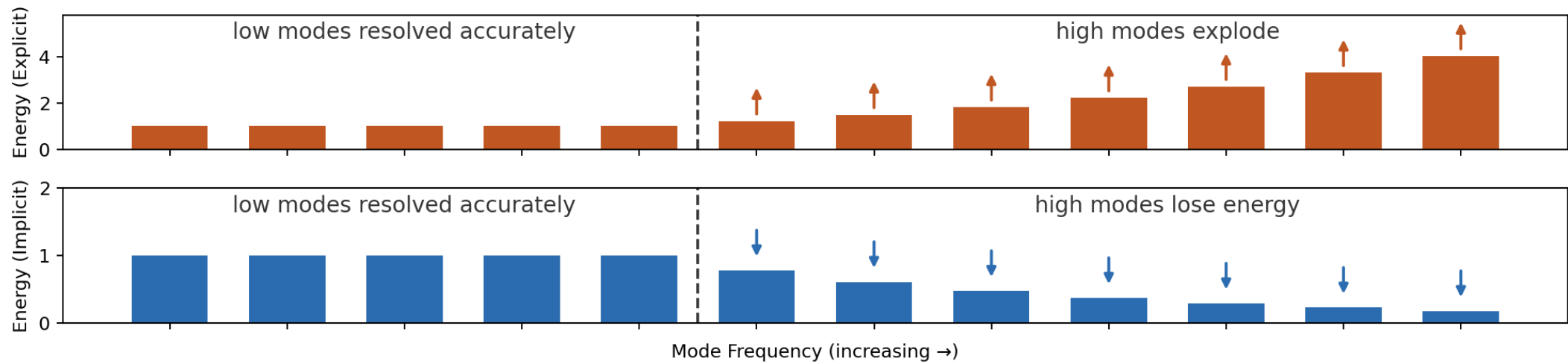
$$f(t + \Delta t) = f(t) + \dot{f}(f(t + \Delta t)) \cdot \Delta t$$

- To compute $f(t + \Delta t)$ you need to already know it. How does that work ???
- Iteratively solve for $f(t + \Delta t)$ -- this is slow.
- Doesn't explode, but it is no more accurate.
- So, why would you do this?

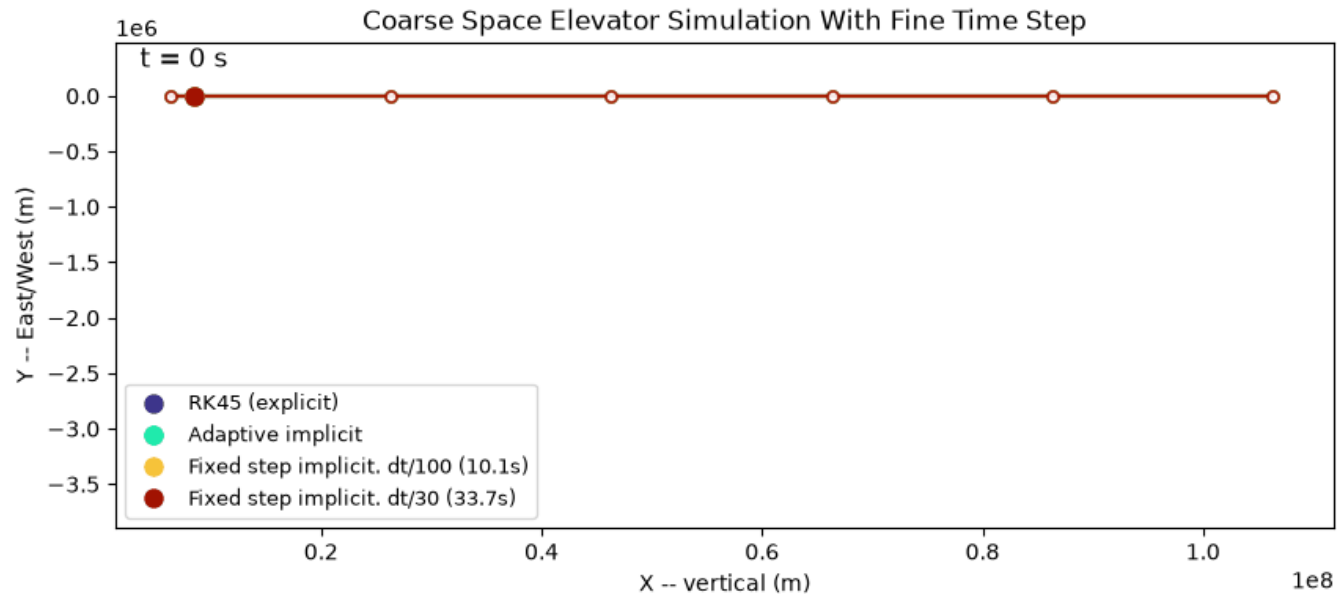


Benefit of Implicit Methods

- Explicit methods force you to use a time step that matches the fastest mode (stiffest degree of freedom). Simulation may be very slow.
- Implicit methods allow you to select the fastest mode you care to model accurately.
- Fast modes will decay and be inaccurate, but will not blow up the simulation.

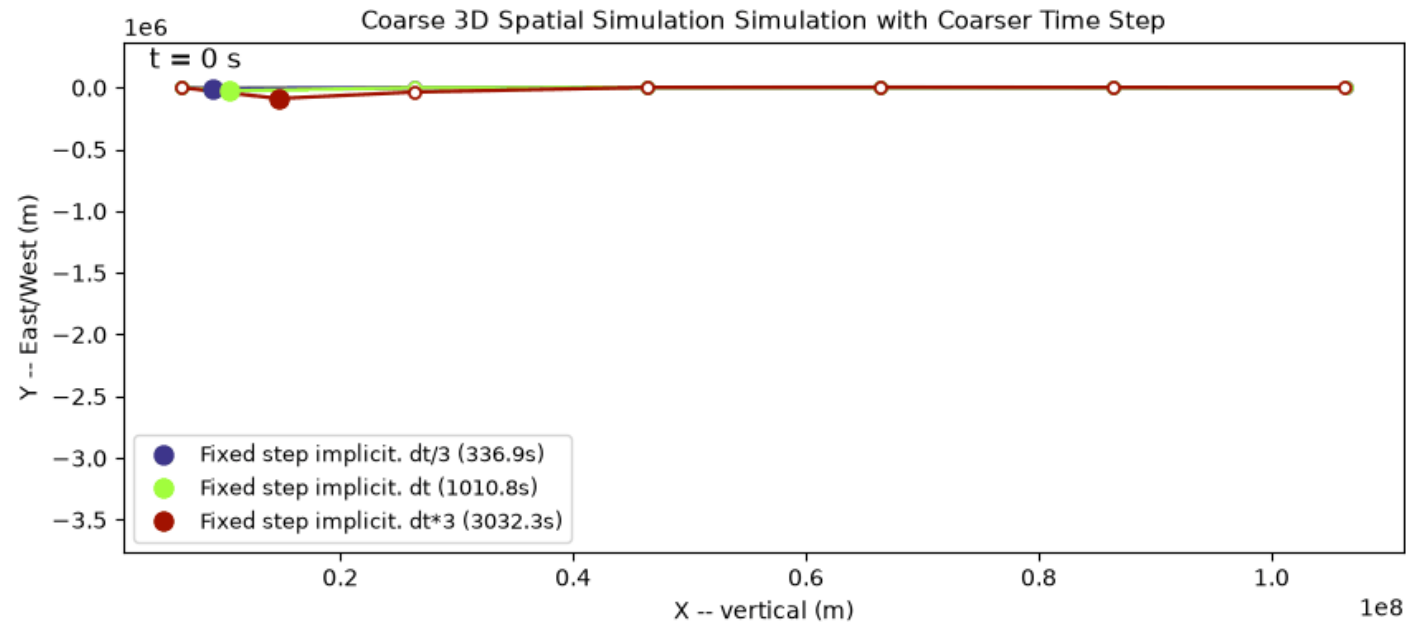


Very Precise Solution of Spatially Coarse Model



- Solve a very coarse 3D SE model using Explicit and Implicit adaptive solvers, as well as with fixed time steps.
- Surprisingly, they all agree on the result, suggesting that all these methods have managed to accurately solve the model, despite the divergence, even using an explicit RK45 method.
- Downward jumps because climber gets deflected to the side as it approaches node.

Coarser Solution



- With larger time steps, the unphysical jumps go away completely! Good agreement over 10x step size variation. Climber hops over nodes, may be good to phase climber motion to voluntarily miss nodes.
- The large step size is heavily damping the diverging mode, allowing us to get a more physically accurate result. **Mystery really solved!** The discretized model was flawed if solved accurately.

Another Benefit of Implicit Methods

- **Benefit 1:** Implicit methods allow simulation of stiff systems without needing small time steps.
- **Benefit 2:** A solver that heavily damps unphysical high frequency modes may produce a solution that better matches the non-discretized system. A perfect integration of the discretized model gives garbage. A less perfect integration gives good results. **Analogy** with image focus:

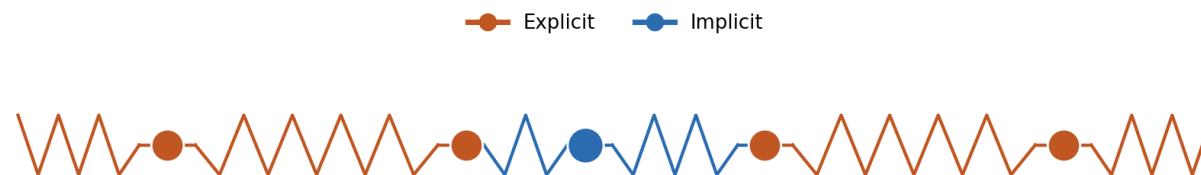


Faster Simulation with Implicit-Explicit (IMEX) Methods

- Split the derivative into two parts, one explicit, one implicit. (Higher order methods are available.)

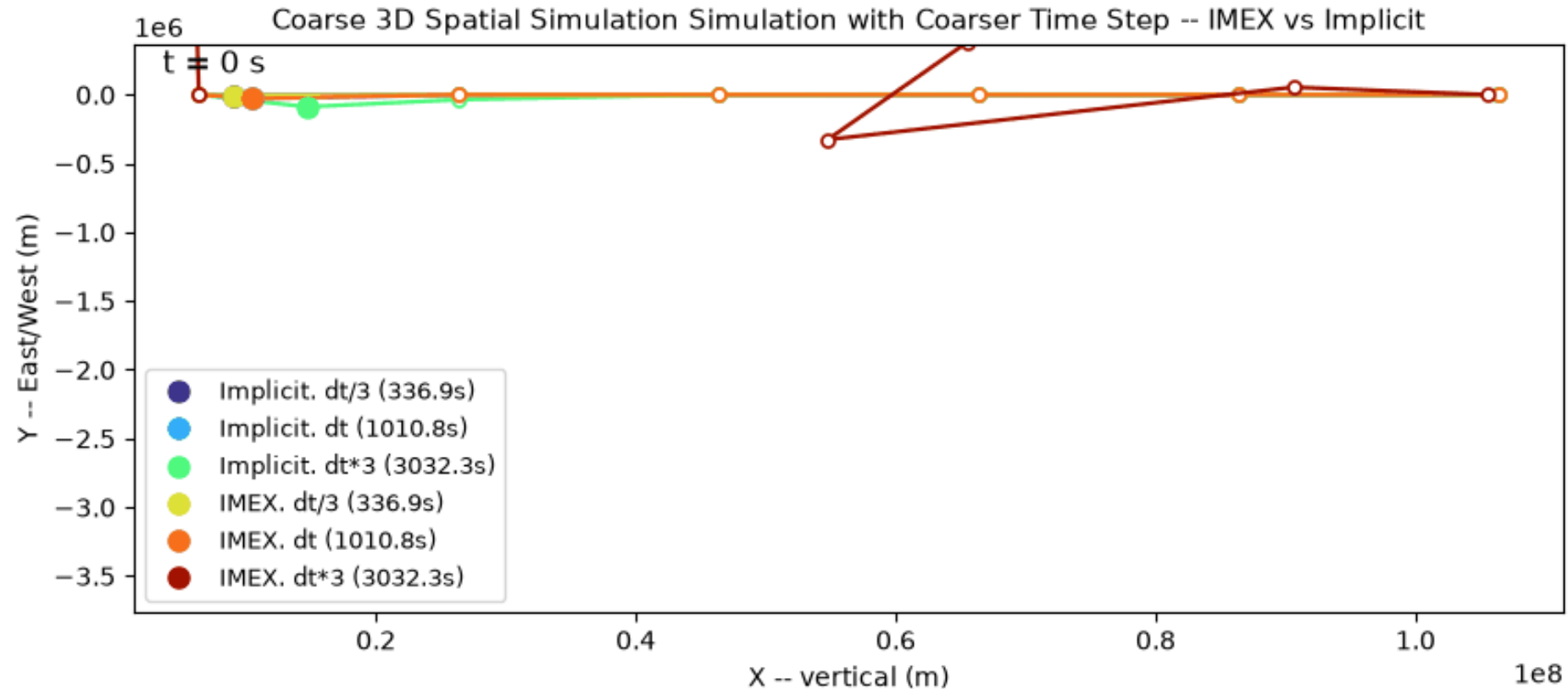
$$\text{IMEX Euler: } f(t + \Delta t) = f(t) + \dot{f}_e(f(t)) \cdot \Delta t + \dot{f}_i(f(t + \Delta t)) \cdot \Delta t$$

- You still need to solve an equation, but if $\dot{f}_i$ is simple, that equation may be fast to solve.
- For the space elevator simulation:
 - climber, all forces acting on it, and their reactions in $\dot{f}_i$
 - everything else in $\dot{f}_e$



- Implicit step equivalent to solving the climber position with the tether frozen in place. Small, fast system of equations.

Imex coarse simulation



- The IMEX method gives the same result as the implicit method, but has more challenges with stability, especially with coarser time steps.

Takeaways

- Climbers approaching tether nodes in a ball and spring simulation break the simulation in two ways:
 - Explicit solvers may go unstable due to the high frequency mode.
 - Exactly solving the model seems to have unphysical behavior.
- So far, we have focused on avoiding these problems by changing the discretization to avoid high stiffness, adding complexity and not always fully correcting the problem.
- Instead we can keep the naive discretization and select a suitable implicit solver.
 - Choose Δt based on the modes that need to be accurate, and the modes that need to be excluded from the simulation.
 - Higher ω modes, which are at best not accurate, and at worst introduce pathological artifacts, will get damped away without harming the simulation.
 - But simulation time may be slow because of large systems of equations to solve repeatedly.
- An IMEX method can be a lower computation load approach to getting rid of the highest frequency mode that causes climber jumps when passing nodes. But you still need to capture all the other high frequency modes so smaller time steps are needed than for fully implicit.

Q&A